DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES

Number and Algebra

Level 2 (Year 3-4)

Teacher Booklet

Task 1	Solve the equations. What do you notice? 30 + 10 = 26 + 20 = 54 + 30 = 39 – 10 = 99 – 10 = Represent your thinking using an empty number line.
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. Number operations and strategies to solve number operations can be recorded using words, numbers, diagrams, and symbols.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-4:** Know how many ones, tens, and hundreds are in whole numbers to at least 1000. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-4:** Know how many tenths, tens, hundreds, and thousands are in whole numbers. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Add and subtract groupings of tens. Represent equations on an empty number line. Use place value to solve addition/subtraction problems.
Mathematical language	Tens, ones, hundreds, place value, face value, total value, base ten.
Sharing back/Connect	Select student solution strategies that focus on the place value and what happens to the tens and ones. Use an empty number line to record adding in tens or larger numbers. Reinforce the language and concepts of nested place value (e.g., Sixty is 6 tens and twenty is 2 tens and 6 tens and 2 tens makes 8 tens or eighty).

	<u>Connect:</u> Ask students to solve the following and use place value language to describe their solutions: 300 + 100 = 360 + 100 = 360 + 20 = 360 + 300 = 360 + 320 =
Teacher Notes	• Before you launch the task, write 147 on the board. Ask students to identify the number and describe it in as may ways as they can. Highlight the place value, face value and total value in the number. • Have concrete material available if needed for students to select (e.g., money in $10 notes and ones, and 100s boards). • Explicitly press for place value and the face and total value. • Expect children to represent their reasoning on an empty number line and track the jumps in either 10's or bigger numbers. If the students do not use these introduce as a representation. • Sets of tens (and tens of tens) can be perceived as single entities e.g. 30 is 3 tens; When we add 40 we are adding 4 tens; 500 is 5 hundreds. Make explicit 30 + 10 is 3 tens plus 1 ten. This highlights the nested nature of place value. Nested place value is the idea that place value units are included in other place value units, for example, tens are within hundreds, and hundreds are within thousands.
Independent Tasks	Solve the following problems: 40 + 20 = 36 + 20 = 55 + 40 = 78 + 10 = 89 - 20 = What patterns do you notice?
Anticipations	

Task 2	What do you notice? 26 + 12 = 54 + 35 = 39 – 15 = Represent your thinking using equations and an empty number line.
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. Number operations and strategies to solve number operations can be recorded using words, numbers, diagrams, and symbols.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-4:** Know how many ones, tens, and hundreds are in whole numbers to at least 1000. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.
Learning Outcomes: Students will be able to:	Add and subtract tens and ones. Name the place, face and total value of numbers. Represent reasoning using a number line and through notation.
Mathematical language	Tens, ones, hundreds, place value, face value, total value.
Sharing back/Connect	Highlight student solution strategies where place value was used. Model how this can be linked to a place value house. **Connect:** Ask students to explain how place value could be used to solve these equations: 115 + 12 = 254 + 34 =
Teacher Notes	• Have concrete material available if needed for students to select (e.g., pre-printed tens frames, money in $10 notes and ones, and 100s boards).

	• Explicitly press for place value and the face and total value, also press for what happens to the ones when you add a ten to a number. • Expect children to represent their reasoning on an empty number line and using equations. • Sets of tens (and tens of tens) can be perceived as single entities e.g. 30 is 3 tens; When we add 40 we are adding 4 tens; 500 is 5 hundreds. Make explicit 30 + 10 is 3 tens plus 1 ten. Highlight the nested nature of place value. Nested place value is the idea that place value units are included in other place value units, for example, tens are within hundreds, and hundreds are within thousands.
Independent Tasks	Solve these problems: 33 + 11 = 23 + 25 = 44 + 35 = 48 – 16 = 56 – 12 = 68 – 25 =
Anticipations	

Task 3	Rena collected 56 stickers. She gave 24 stickers to her friend. How many stickers does Rena have left? Kelly collected 99 stickers. She gave 73 stickers to her friend. How many stickers does Kelly have left? Asa collected 168 stickers. She gave 31 stickers to her friend. How many stickers does Asa have left?
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. Numbers are used to name specific quantities. Numbers can be decomposed into parts in an infinite number of ways without the quantity changing. Number operations and strategies to solve number operations can be recorded using words, numbers, diagrams, and symbols.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-4:** Know how many ones, tens, and hundreds are in whole numbers to at least 1000. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Use place value to solve subtraction problems. Represent reasoning using a number line and through notation.
Mathematical language	Tens, ones, hundreds, subtract.
Sharing back/Connect	Select student solution strategies that have used place value: *168 = 100 + 60 + 8 so to take away 31 …* *100 – 0 = 100* *60 – 30 = 30* *8 – 1 = 7* *168 – 31 = 137* *or*

	168 – 30 = 138 *138 – 1 = 137* Ask students to represent or model how to represent using both equations and an empty number line **Connect:** Ask students to explain how you would solve the following equations using place value: 78 – 34 = 165 – 42 =
Teacher Notes	• Have concrete material available if needed for students to select (e.g., pre-printed tens frames, money in $10 notes and ones, and 100s boards). • Expect students to represent their reasoning on an empty number line and track the jumps in either 10's or bigger numbers. If the students do not use these introduce their use as a representation. • Also expect students to use equations to represent their reasoning. Model this if students do not use equations. • Notice students who are subtracting by using tens and ones.
Independent Tasks	May-Lee had 88 beads. She made a necklace for her cousin and used 56 beads. How many beads does May-Lee have left over? Beth had 37 beads. She made a necklace for her cousin and used 21 beads. How many beads does Beth have left over? Dakota had 56 beads. She made a necklace for her cousin and used 32 beads. How many beads does Dakota have left over?
Anticipations	

Task 4	Monty is sorting his blocks into colours. He has 56 blue blocks and 19 red blocks. How many blocks does Monty have altogether? Jason is sorting his blocks into colours. He has 39 yellow blocks and 38 white blocks. How many blocks does Jason have altogether? Pauli is sorting his blocks into colours. He has 27 black blocks and 25 orange blocks. How many blocks does Pauli have altogether?
Big Ideas	Numbers are used to name specific quantities. Numbers can be decomposed into parts in an infinite number of ways without the quantity changing. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-4:** Know how many ones, tens, and hundreds are in whole numbers to at least 1000. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Use place value to solve addition problems. Use bridging by decades to solve addition problems. Use equivalence and compensation to solve addition problems. Represent reasoning using a number line and through notation.
Mathematical language	Tens, ones, add, subtract.
Sharing back/Connect	Select student solution strategies that have bridged across a decade or used equivalence and compensation. If no student solves the task this way, then introduce either solution strategy as an alternative model previously used by other students. Use multiple representations to represent student solution strategies

	including an empty number line, equations, and tens frames. *Bridging across tens* *39 + 38 =* *39 + 1 = 40* *40 + 30 = 70* *70 + 7 = 77* *Equivalence and compensation* *39 + 38 =* *39 + 40 = 79* *79 – 2 = 77* **Connect:** Ask students to describe how you would solve the following equations using either bridging across a decade or equivalence and compensation: 58 + 19 = 155 + 27 =
Teacher Notes	• Have concrete material available if needed for students to select (e.g., pre-printed tens frames, money in $10 notes and ones, and 100s boards). • Notice students who are adding the numbers by bridging to the closest decade. • Notice students who are using equivalence and compensation. • Expect students to represent using an empty number line and equations.
Independent Tasks	Dallas and Vegas collected acorns in the playground. Dallas has 18 acorns and Vegas has 46. How many acorns do they have altogether? Roman and Elias collected acorns in the playground. Roman had 59 acorns and Elias had 38. How many acorns do they have altogether? Jack and Jill collected acorns in the playground. Jack has 65 acorns and Jill has 27. How many acorns do they have altogether?
Anticipations	

Task 5	Manu was helping at the sausage sizzle. They cooked 56 sausages and sold 18 sausages. How many sausages do they have left over? Sepi was helping at the sausage sizzle. They cooked 32 sausages and sold 19 sausages. How many sausages do they have left over? Don was helping at the sausage sizzle. They cooked 74 sausages and sold 17 sausages. How many sausages do they have left over?
Big Ideas	Numbers are used to name specific quantities. Numbers can be decomposed into parts in an infinite number of ways without the quantity changing. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-4:** Know how many ones, tens, and hundreds are in whole numbers to at least 1000. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Use subtracting in parts to solve subtraction problems. Use equivalence and compensation to solve subtraction problems. Represent reasoning using a number line and through notation.
Mathematical language	Tens, ones, add, subtract.
Sharing back/Connect	Notice and select student solution strategies where they have subtracted in parts or used equivalence and compensation. Represent this using equations and on an empty number line. *Subtraction in parts*

	56 – 18 = *56 – 10 = 46* *46 – 6 = 40* *40 – 2 = 38* *Equivalence and compensation* *56 – 18 =* *56 – 20 = 36* *36 + 2 = 38* **Connect:** Ask students to describe how you would solve the following equations using either subtracting in parts or equivalence and compensation: 52 – 18 = 86 – 29 = Use an empty line and equations to represent their ideas.
Teacher Notes	• Have concrete material available if needed for students to select (e.g., pre-printed tens frames, money in $10 notes and ones, and 100s boards). • Students may either subtract in parts or use equivalence and compensation (take away more and then adjust proportionally). • Students may solve using inverse relationships. Use this to highlight the relationship between addition and subtraction. • Expect students to represent using equations and empty number lines.
Independent Tasks	Ana had 41 loom bands. She used 17 to make a bracelet for her friend. How many loom bands does she have left? Tina has 28 loom bands. She used 19 to make a bracelet for her friend. How many loom bands does she have left? Sam has 52 loom bands. She used 26 to make a bracelet for her friend. How many loom bands does she have left?
Anticipations	

Task 6	Nana had 16 plums. She asked Sela to pick some more plums. Now she has 45 plums. How many plums did Sela pick? Aunty had 24 feijoas. She asked Tama to pick some more. Now Aunty has 51 feijoas. How many feijoas did Tama pick? Dad counted 44 nails in his toolbox. He found some more in the garage. Now he has 72 nails. How many nails did Dad find in the garage?
Big Ideas	Numbers are used to name specific quantities. Numbers can be decomposed into parts in an infinite number of ways without the quantity changing. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-4:** Know how many ones, tens, and hundreds are in whole numbers to at least 1000. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Use subtracting in parts to solve subtraction problems. Use equivalence and compensation to solve subtraction problems. Use the inverse relationship of addition and subtraction to solve problems. Represent reasoning using a number line and through notation.
Mathematical language	Tens, ones, add, subtract, inverse relationship.
Sharing back/Connect	Select a student solution strategy which uses addition and a different one which has used subtraction. Facilitate students to compare the solution strategies and use this to highlight the inverse relationship between addition and subtraction. If no student solves the task this

	way, then introduce either solution strategy as an alternative model previously used by other students. **Connect:** Ask students to represent the following situation using at least two different equations and operations: Ana had 61 mandarins. She keeps some mandarins but gives 16 to her neighbour. How many mandarins did Ana keep? Niko had 24 plums in a bowl. He collected some more plums and now he has 73 plums. How many plums did Niko collect?
Teacher Notes	• Launch these tasks by asking students to act out the scenario so that they can access the structure of the tasks. • Have concrete material available if needed for students to select (e.g., pre-printed tens frames, money in $10 notes and ones, and 100s boards). • Students may draw on the inverse and solve using addition or subtraction. • Focus on supporting students to represent using equations and an empty number line to solve the tasks.
Independent Tasks	Uncle had 22 feijoas in one bag and 14 feijoas in another bag. How many feijoas does uncle have altogether? Mona picked 54 strawberries and her cousin picked some more. Now they have 92 strawberries. How many did cousin pick? Mere has 37 pink beads in one bag. She also has some yellow beads in another bag. Altogether she has 76 beads. How many yellow beads does she have?
Anticipations	

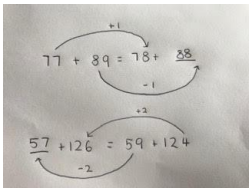
Task 7	Solve the following problems: 56 – 17 = 83 – 38 = 54 + 37 = 139 + 26 =
Big Ideas	Numbers are used to name specific quantities. Numbers can be decomposed into parts in an infinite number of ways without the quantity changing. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Use subtracting in parts to solve subtraction problems. Use place value to solve addition problems. Use bridging to the nearest decade to solve addition problems Use equivalence and compensation to solve subtraction and addition problems. Represent reasoning using a number line and through notation.
Mathematical language	Tens, ones, add, subtract.
Sharing back/Connect	Select student solution strategies where they have subtracted in parts, used place value to add or used equivalence and compensation. **Connect:** Ask students to describe how you would solve the following equations

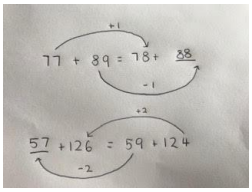
	using place value, bridging to the nearest decade, subtraction in parts or equivalence and compensation: 73 – 24 = 39 + 163 =
Teacher Notes	• Have concrete material available if needed for students to select (e.g., pre-printed tens frames, money in $10 notes and ones, and 100s boards). • Notice students who are adding the numbers by bridging to the closest decade. • Notice students who are using equivalence and compensation. • Expect students to represent using the empty number line and equations.
Independent Tasks	Solve the problems below: 63 – 19 = 45 – 26 = 82 – 57 = 43 + 118 = 112 + 98 = Use an empty number line and equations to show how you have solved them.
Anticipations	

Task 8	Solve the following problems: 94 – 37 = 153 – 49 = 121 – 68 =
Big Ideas	Numbers are used to name specific quantities. Numbers can be decomposed into parts in an infinite number of ways without the quantity changing. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Use subtracting in parts to solve subtraction problems. Use equivalence and compensation to solve subtraction problems. Represent reasoning using a number line and through notation.
Mathematical language	Hundred, tens, ones, add, subtract.
Sharing back/Connect	Select student solution strategies where they have subtracted in parts or used equivalence and compensation. Represent this with equations and on an empty number line. **Connect:** Ask students to describe how you would solve the following equations using either subtraction in parts or equivalence and compensation:

	165 – 59 = 113 – 87 =
Teacher Notes	• Provide materials to help students who need it access the maths. • Notice students who are adding the numbers by bridging to the closest decade. • Notice students who are using rounding and compensating. • Expect students to represent using the empty number line and equations. Also have printed or empty tens frames available for students to model their solution strategy.
Independent Tasks	Solve the following problems: 83 – 65 = 161 – 29 = 183 – 69 = 85 – 47 = Represent your thinking using equations and on an empty number line.
Anticipations	

Task 9 (Whole class option)	Are these number sentences true or false? Justify your thinking. 188 = 188 99 + 255 = 256 + 99 78 = 50 + 28 29 + 30 = 59 + 8 34 – 19 = 33 – 18 250 = 261 28 + 26 = 29 + 25
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Explain and justify relationships between numbers in an equation. Write statements of equivalence in words and using notation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equal sign, relationship.
Sharing back/Connect	Allow students to share misconceptions related to the equal sign to position them to engage in argumentation. Draw out discussion that the equal sign means the same on both sides but also that it shows a relationship. Select students to share who have used a relational strategy to recognise equivalence. *"28 + 26 = 29 + 25 because you add one to 28 to get 29 so you need*

	to take one away from 26 to make it 25" **Connect:** Ask students to write their own true and false number sentences. Note students who use the equal sign flexibly.
Teacher Notes	• Before you launch the task, ensure that students understand what true and false means. Introduce notation of not equal (≠) for the number sentences that they think are false • Students may begin by demonstrating misconceptions (29 + 30 = 59 + 8 is true because 29 + 30 = 59). This can be used to position students to agree/disagree. • Some students may work out one side and then the other to equal the same number. However, the key focus should be on positioning students to use the relationships across the equal sign. • Draw attention to students who use relational types of thinking and notate the number sentences with arrows to highlight this. 
Independent Tasks	Solve the following problems: 126 + 57 = 122 – 72 = 74 + 168 = 137 + 85 = 192 – 65 =
Anticipations	

Task 10	Can you find the missing numbers? 8 + 6 = __ + 5 35 + 19 = 36 + __ 19 + __ = 17 + 25 __ + 86 = 19 + 85
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Explain and justify relationships between numbers in an equation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equal sign, relationship, difference, add, subtract.
Sharing back/Connect	Allow students to share misconceptions related to the equal sign (e.g., 8 + 6 = 14 + 5) to position them to engage in argumentation. Select students to share who have used a relational strategy to find the missing number. If no students use a relational strategy, introduce this to them using arrows and explanations. 

	Connect: Ask the students to find the missing numbers by looking for the relationship across the equal sign and show this using arrows. 77 + 89 = 78 + __ __ + 126 = 59 + 124
Teacher Notes	• Students may begin by demonstrating misconceptions. This can be used to position students to agree/disagree. • Some students may work out one side and then the other to equal the same number. However, the key focus should be on positioning students to use the relationships across the equal sign. • Draw attention to students who use relational types of thinking and notate the number sentences with arrows to highlight this.
Independent Tasks	Are these number sentences true or false? Be ready to explain and justify your thinking. 76 = 86 – 10 118 + 47 = 48 + 117 96 – 5 – 2 = 96 – 7 176 – 87 = 177 – 88 15 + 18 = 14 + 17
Anticipations	

Task 11	Find the missing numbers: 23 – 17 = __ – 15 46 – 28 = 45 – __ __ – 36 = 71 – 26 143 – __ = 43 – 29
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA2-7:** Generalise that whole numbers can be partitioned in many ways. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Explain and justify relationships between numbers in an equation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equal sign, relationship, difference, add, subtract.
Sharing back/Connect	Select students to share who have used a relational strategy to find the missing number. Notate the equations using arrows. **Connect:** Ask the students to find the missing numbers by looking for the relationship across the equal sign and show this using arrows. 34 – 17 = 35 – __ __ – 89 = 126 – 99
Teacher Notes	• Note that the order of directionality is different between addition and subtraction and students may adjust as you do with addition and end up with an incorrect solution such as 23 – 17 =

	25 – 15. Facilitate a discussion with the students to notice the difference between open number sentences with addition and subtraction (e.g., addition involves an adjustment of +1, -1 while subtraction involves an adjustment of +1, +1, or -1, -1). • Some students may work out one side and then the other to equal the same number. However, the key focus should be on positioning students to use the relationships across the equal sign. • Draw attention to students who use relational types of thinking and notate the number sentences with arrows to highlight this.
Independent Tasks	Find the missing numbers: 18 + 15 = __ + 16 __ + 27 = 14 + 29 22 – __ = 23 – 17 64 – 38 = 62 – __ 157 + 178 = __ + 168 __ – 67 = 253 – 167 Use arrows to show your thinking.
Anticipations	

Task 12 (Whole class option)	Jonty said "When you are adding two numbers together, it doesn't matter which order you use to add them, the answer will be the same". Work in a group and explore whether you agree or disagree with this statement. Can you prove that it works for all numbers? Does what Jonty said also work for subtraction, multiplication, and division?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Explain and justify the commutative property. Use different representations including concrete material, representations, and notation to represent a conjecture.
Mathematical language	Commutative property, conjecture, proof, generalisation, addition, subtraction, multiplication, division.
Connect	Select groups that have built concrete models to share their generalisations. Highlight to students that letters or symbols can be used in maths to represent any numbers. **Generalise:** Can you represent the conjectures that you have made using a statement, a diagram and a number sentence (e.g., $\Delta + \square = \square + \Delta$)?
Teacher Notes	• Students may begin by testing different examples with numbers and different types of numbers (e.g., large, small, fractions). After they have explored multiple examples, prompt them by

	asking whether they can prove it would work with **every** number. • Have appropriate equipment for students to build concrete models to prove their conjectures (e.g., counters, grid paper, peg boards). • Look for students drawing on the commutative property and understanding that it works for addition and multiplication but not for subtraction and division. • Students may generate counter examples to prove the commutative property does not apply to subtraction or division. Students may also generate special cases (e.g., 5 – 5 = 5 – 5).
Independent Tasks	Find the missing numbers: 46 – 18 = __ – 16 67 – 49 = __ – 48 __ – 25 = 63 – 15 193 – __ = 93 – 29
Anticipations	

Task 13 (Whole class option)	If 136 + 287 = 423 Then 287 + 136 = 423 – 136 = 423 – 287 = What patterns do you notice? Does this always work? Write your own sets of addition and subtraction equations that use the same patterns.
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Explain and justify the commutative property. Explain and justify the inverse relationship of addition and subtraction.
Mathematical language	Commutative property, inverse relationship, generalisation, addition, subtraction.
Sharing back/Connect	Select student solution strategies that draw on understanding the inverse relationship of addition and subtraction rather than calculating answers. Highlight to the students that you do not need to calculate but can use the relationship to generate different equations. Ask students to consider whether this will always work and when it will not work. For example: 23 + 18 = 41 23 – 41 ≠ 18

	Connect: If 2387 + 4894 = 7281 Can you write three 'then' addition and subtraction equations that are true using the same numbers? If $a + b = c$ Can you write three 'then' addition and subtraction equations that are true
Teacher Notes	• Addition and subtraction are inverse operations. • Some students will want to calculate answers, support them to look for relationships and connections across operations.
Independent Tasks	Solve the following problems: 27 + 125 = 156 – ___ = 13 ___ + 72 = 188 13 + 155 = 50 = ____ + ____ 46 = _____ + _____ 52 + 19 = 36 + 152 =
Anticipations	

Task 14 (Whole class option)	The following number sentence is true: 72 – 57 = 15 Is 72 – 57 – 8 = 15 – 8 true or false? Is 72 – 57 + 36 = 15 + 34 true or false? How do you know? Do you think this will work for other numbers? Can you explain why or why not? Can you write your own examples with other numbers where this relationship works?
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA2-1:** Use simple additive strategies with whole numbers and fractions. **NA2-6:** Communicate and interpret simple additive strategies using words, diagrams (pictures) and symbols. **NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers.
Learning Outcomes: Students will be able to:	Explain and justify that the equals sign means equivalence or balance. Use relational thinking within number operations.
Mathematical language	Equal sign, relationship, conjecture, difference, add, subtract.
Sharing back/Connect	Select student solution strategies that draw on understanding that if you subtract or add the same number on both sides of the equal sign the equation will remain correct rather than calculating the answers. Highlight to the students that you do not need to calculate but can use the relationship across the equal sign. Ask students to consider whether this will always work and when it will not work.

	Connect: Use quasi-variables (large numbers acting as a variable) to press the students to generalise relationships. For example: If 186 + 189 = 375 Then is 186 + 189 + 1999 = 375 + 1999 true or false? Ask students to generate a conjecture about this pattern 'if you add the same number on both sides of the equal sign, then the two sides will be equal". Ask students to represent this using an 'if and then' statement: If $a + b = c$ Then $a + b + d = c + d$
Teacher Notes	• Students may compute each sum separately or notice that subtracting or adding the same number on both sides of the equal signs will result in balance. • Draw student attention to those who notice the relationship across the equal sign. • Allow students the opportunity to explore the relationships and also encourage them to explore this with subtraction. • A quasi-variable is a large number that can represent any number. Students do not need to solve these examples, rather they look at the relationships and use that to explain what they notice/ what is happening mathematically.
Independent Tasks	Select one or more of the following assessment tasks (attached at the end of the document) as the independent activity: N1A: Addition and subtraction problems to solve. N3A: Properties of numbers and operations.
Anticipations	

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DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

NUMBER - ADD / SUB: LEVEL 2 Task N1A

Georgia has 27 stickers in her collection. She has a sheet with another 38 stickers. How many stickers does Georgia have altogether? Prove and justify your answer.

Hamuera is playing marbles. He has 53 marbles but loses 25 marbles in the game. How many marbles does Hamuera have now? Prove and justify your answer.

Tatiana's rugby team scored 243 points over the season. They scored 86 points more than the next team in the league. What did the next team score? Prove and justify your answer.

Write one or more word problems for a friend involving addition or subtraction. Show how you would solve it.

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DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASKS

NUMBER - PATTERNS: LEVEL 2 Task N3A

65 + 38 = 89 + 26 = 17 + 45 + 23 = 11 x 7 =

38 + 65 = 77 ÷ 7 = 90 + 25 =

11 + 11 + 11 + 11 + 11 + 11 + 11 = 20 + 30 =

7 x 11 = 50 – 20 = 23 + 17 + 45 =

Look at the number sentences above.

- Describe what patterns you can find
- Why do your patterns work?
- Do they work with other numbers?