DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES

Number and Algebra

Level 3 (Year 5-6)

Teacher Booklet

Task 1	What do you notice? Justify your thinking. 246 + 372 = 2561 + 339 = 6 331 + 1899 =
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
Curriculum Links	**NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-2:** Know basic multiplication and division facts. **NA3-4:** Know how many tenths, tens, hundreds, and thousands are in whole numbers. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers
Learning Outcomes Students will be able to:	Explain the face, place, and total value of the digits in numbers. Explain and justify the use of place value to solve addition problems. Explain and justify the use of equivalence and compensation to solve addition problems. Represent equations on an empty number line, in notation and using a place value house.
Mathematical language	Ones, tens, hundreds, thousands, add, subtract, place value, face value, total value, digit.
Sharing back/Connect	Select student solution strategies which use a form of place value or equivalence and compensation to solve the problems. Model the student solution strategy using place value houses and an empty number line.

	Place value *2561 + 339 = 2000 + (500 + 300) + (60 + 30) + (1 + 9)* *Equivalence and compensation* *2561 + 339 =* *2561 + 340 = 2901* *2901 – 1 = 2900* **Connect:** Ask the students to explain how you would solve the following equations using place value or equivalence and compensation: 246 + 3258 = 6234 + 2863 =
Teacher Notes	• Before you launch the task, write 1629 on the board. Ask students, what is this number? How can you write and explain this number in different ways? Support the students to read the number correctly. Give them an opportunity to work in pairs and record and represent their reasoning. Explore concepts of place, face, and total value. • Support students to discuss thousands, hundreds, tens, ones and make links to the place value houses and place, face, and total value. • Have a place value house on the wall or whiteboard for students to refer to.
Independent Tasks	Solve these equations: 246 + 39 = 236 + 3782 = 5 232 + 2 989 =
Anticipations	

Task 2	Moana is playing Monopoly with her friends. She had $235 in monopoly money. After she had bought two properties, she had only $119 in monopoly money left. How much did she spend to buy the two properties? Moana is playing Monopoly with her friends. She had $1327 in monopoly money. After she had bought five properties, she had only $158 in monopoly money left. How much did she spend to buy the five properties?
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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Mathematical language	Ones, tens, hundreds, thousands, add, subtract, place value, face value, total value, digit, addition, subtraction, inverse relationship.

Sharing back/Connect	Select student solution strategies that have used inverse relationships of addition and subtraction, equivalence and compensation or place value and renaming. *Inverse relationship* *235 – 119 =* *119 + __ = 235* *Equivalence and compensation* *235 – 119 =* *235 – 120 = 115* *115 + 1 = 116* *Place value and renaming* *235 – 119 =* *5 – 9 = ?* *Rename one ten so 15 ones – 9 ones = 6* *20 – 10 = 10* *200 – 100 = 100* **Connect:** Ask students to solve the following equations and describe any patterns they notice: 100 – 49 = 1000 – 449 = 10000 – 4449 =
Teacher Notes	• Notice use of place value and the ability to see hundreds as ten tens and tens as ten ones. Draw connections to represent these within place value houses. • Introduce empty number line as a way to represent solution strategies. • Expect students to use equations to represent their thinking.
Independent Tasks	Solve the following equations: 327 – 152 = 442 – 374 = 8 222 – 5 768 =
Anticipations	

Task 3	Mere and Hurae are playing the Game of Life. Hurae wins the golden lottery and now has $7442. Before he won the golden lottery, he had $2789. How much money did he win? Mere and Hurae are playing the Game of Life. Hurae wins the golden lottery and now has $5432. Before he won the golden lottery, he had $4785. How much money did he win?
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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Mathematical language	Ones, tens, hundreds, thousands, add, subtract, place value, face value, total value, digit, addition, subtraction, inverse relationship.
Sharing back/Connect	Select student solution strategies that have used inverse relationships of addition and subtraction, equivalence and compensation or place value and renaming.

	Connect: Ask students to describe what they notice is similar and different in the student solution strategies. Will the solution strategy always work?
Teacher Notes	• Notice use of place value and the ability to see hundreds as ten tens and tens as ten ones. Draw connections to represent these within the place value houses. • Expect students to use equations and the empty number line to represent their thinking. • If students use the standard algorithm, ensure procedural understanding.
Independent Tasks	Solve the following equations: 535 – 266 = 434 – ☐ = 216 ☐ – 539 = 182 2 544 – 1 689 =
Anticipations	

Task 4	Junior's mum needs to order tipani flowers to make 'ei katu for his sister's wedding. There are 19 people in the bridal party and each 'ei katu needs 26 tipani flowers. How many flowers will Junior's mum need to order? Junior's mum needs to order tipani flowers to make 'ei katu for his sister's wedding. There are 18 people in the bridal party and each 'ei katu needs 22 tipani flowers. How many flowers will Junior's mum need to order?
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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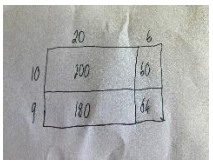
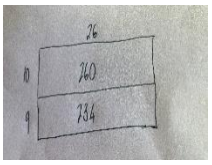
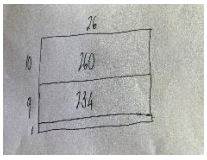
Learning Outcomes Students will be able to:	Explain and justify the use of the distributive property in multiplication. Explain and justify the use of equivalence and compensation in multiplication. Represent reasoning using different forms of notation including an area and an array model.
Mathematical language	Distributive property, area, equivalence, compensation, factor, product.
Sharing back/Connect	Select student solution strategies which use the distributive property or equivalence and compensation.

	Distributive property *26 x 19 = (26 x 10) + (26 x 9)* *26 x 19 = (20 x 10) + (20 x 9) + (6 x 10) + (6 x 9)* *Equivalence and compensation* *26 x 19 = (26 x 20) – (26 x 1)* If either solution strategy has not been used, introduce this as a solution strategy that students have used previously. Record these as equations and model representing these using the area model.    **Connect:** Ask students to describe how you would solve the following equation using either the distributive property or equivalence and compensation and represent it using the area model: 34 x 29 =
Teacher Notes	• Notice student solution strategies either using distributive property or equivalence and compensation. Explicitly talk about the type of mathematical property they have used and use correct mathematical language. • Introduce students to representations using array/area model. • Expect students to record their solutions using equations.
Independent Tasks	Solve the following equations: 17 x 23 = 29 x 21 = 38 x 37 = Represent your solution strategy using equations and an area model.
Anticipations	

Task 5	Nga and her family are planning a family reunion. It is Nga's job to look at what funding is needed for this and she needs make an accurate estimate for fundraising. Nga says that there are 284 people coming including children and that $36 per person should cover the costs for them all. How much do they have to fundraise? What if they had to raise or lower the cost? Explore whether your solution strategy would work with other possible amounts.
Big Ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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Learning Outcomes Students will be able to:	Explain and justify the use of the distributive property in multiplication. Explain and justify the use of the associative property in multiplication. Represent reasoning using different forms of notation including an area and an array model.
Mathematical language	Distributive property, area, associative property, factor, product.

Sharing back/Connect	Select and sequence student solution strategies that use the distributive property or associative property. *Associative property* *284 x 36 = (284 x 3 x 10) + (284 x 6)* If either solution strategy has not been used, introduce this as a solution strategy that students have used previously. **Connect:** Ask students to describe how the associative property would be used if multiplying by 20 or 50. Ask students to describe how the equation below could be solved by using either the distributive and/or associative property: 245 x 123 = Model links to standard written algorithm for multiplication (if appropriate).
Teacher Notes	• Expect students to record using equations and the area model. • Notice students' solution strategies using the distributive property or the associative property. • Explore what happens when using the associative property. • If students use the standard algorithm, links could be made between this and the distributive property.
Independent Tasks	Solve the following equations: 145 x 56 = 236 x 471= 3869 x 525 = 6798 x 9825 = What patterns did you notice and use to help you solve the equations? Would the patterns work for any numbers when multiplying?
Anticipations	

Task 6	At Polyfest, there are 278 dancers in the Sāsā group. If they sit in rows of 15, how many rows will there be? Will there be some people left over to make back row which is not the same size as the front rows? What possible numbers would they have to use to get the exact numbers in every row and with no people left over? Make sure you can prove this using an example which you can explain and justify.
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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Learning Outcomes Students will be able to:	Explain and justify the use of the partial quotients/distributive property in division. Explain and represent the inverse relationship of multiplication and division. Represent reasoning using different forms of notation.
Mathematical language	Distributive property, inverse relationship, factor, product, quotient, divisor, dividend.
Sharing back/Connect	Select student solution strategies where they have used the inverse relationship of multiplication and division or the partial quotient/distributive property in the solution. If either solution strategy has not been used, introduce this as a solution strategy that students have used previously.

	Inverse relationship *278 ÷ 15 =* *15 x ? = 278* *15 x 10 = 150 …* *Distributive property/partial quotients* *278 ÷ 15 = (150 ÷ 15) + (60 ÷ 15) + (60 ÷ 15) + (8 ÷ 15)* **Connect:** Ask students to describe how you would solve the following equation using either the inverse relationship or the partial quotient/distributive property: 487 ÷ 35 =
Teacher Notes	• Select strategies that start at use of some form of multiplicative thinking. • If addition or subtraction used have students rework as multiplication or division. • Notice whether students draw on multiplying by ten when using the inverse relationship. Model use of x 10 then x 5 as an easy process. • Notice whether students have used partial quotients . • Note use of doubling and shift towards concept of multiplying by two as doubling.
Independent Tasks	Solve the following equations: 556 ÷ 25 = 866 ÷ 42 = 765 ÷ 33 =
Anticipations	

Task 7	Our school is going on a picnic and using buses to take all the children, teachers, and adults. Each bus can take 46 passengers and there are 942 people to transport. How many buses do we need? What numbers could you use with your solution strategy that would mean you had the same number of people in every bus? Be ready to explore and explain at least three other sets of numbers.
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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Learning Outcomes Students will be able to:	Explain and justify the use of the partial quotients/distributive property in division. Explain and represent the inverse relationship of multiplication and division. Represent reasoning using different forms of notation.
Mathematical language	Distributive property, inverse relationship, factor, product, quotient, divisor, dividend.
Sharing back/Connect	Select student solution strategies where they have used the partial quotient/distributive property in the solution. **Connect:** What numbers would you change these into when using partial quotient/distributive property to divide?

	776 ÷ 35 = 867 ÷ 42 = 935 ÷ 31 = Have children discuss possible number combinations without solving these. Model links to the relationship between the partial quotients/distributive property and the standard division algorithm.
Teacher Notes	• Notice students who are using addition or subtraction and support them to re-work as multiplicative thinking. • Notice students who use the inverse property or who are using partial quotients/distributive property in their calculations.
Independent Tasks	Solve the following equations: 387 ÷ 49 = 822 ÷ 73 = 778 ÷ 86 = $1 \div \frac{1}{2} =$ $2 \div \frac{1}{4} =$
Anticipations	

Task 8	The library needs to be packed up to be moved. There are 2953 books that need to be packed and each box will fit 187 books. How many boxes are needed?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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Mathematical language	Distributive property, inverse relationship, factor, product, quotient, divisor, dividend.
Sharing back/Connect	Select student solution strategies where they have used the inverse relationship in the solution. **Connect:** Ask students to explain and justify the inverse relationship of multiplication and division and discuss what they would multiply by to estimate the answers to the division problems 4897 ÷ 243 = 4625 ÷ 2251 =

Teacher Notes	• Notice students using the inverse relationship. Support them to notice the efficiency of multiplication by 10. • Expect students to represent using equations.
Independent Tasks	Solve the following equations: 7085 ÷ 385 = 8643 ÷ 221 = 9999 ÷ 2133 = $\frac{1}{2} \div \frac{1}{4} =$
Anticipations	

Task 9 (Whole class option)	Work in your group to decide which number sentences are true or false? Make sure you prove and explain your reasoning. 188 = 188 99 + 255 = 255 + 99 45 – 17 = 43 – 15 37 = 10 + 26 38 + 26 = 39 + 25 45 – 7 = 38 – 5
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
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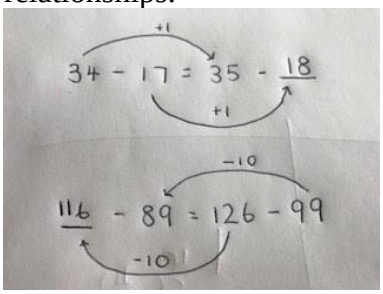
Learning Outcomes Students will be able to:	Explain and justify relationships between numbers in an equation. Write statements of equivalence in words and using notation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equivalent, equal sign, commutative property.
Sharing back/Connect	Allow students to share misconceptions related to the equal sign (e.g., 8 + 6 = 14 + 5) to position them to engage in argumentation. Select student solution strategies that use relational reasoning. *45 – 17 = 43 – 15 is true because 43 is two less than 45 and 15 is two less than 17.* If no students use a relational strategy, model this to them using arrows and explanations. **Connect:** Can you work out whether the following are true or false without calculating each side?

	472 – 449 = 72 - 49 117 + 239 = 127 + 249
Teacher Notes	● Ensure that students understand what true and false means. Introduce notation of not equal (≠) for the number sentences that they think are false. ● Students may initially treat the equals sign as an operator or indication to write the answer next. These misconceptions can be used to position students to engage in mathematical argumentation. ● Students also may compute each side to work out whether they are equal. ● Notice students who use the relationships across the equals sign to see whether there is balance. ● Highlight the students' relational responses (e.g., noticing the + 2, – 2 relationships). ● Press for use of arrows and notations to highlight the relationships. 
Independent Tasks	Explain and justify which number sentences are true and false: 19 = 1 + 8 + 10 15 + 17 = 16 + 18 225 – 178 = 235 – 168 25 – 5 = 20 – 2 183 – 87 = 181 – 89 5 + 18 + 87 = 6 + 17 + 87
Anticipations	

Task 10 (Whole class option)	Work together to decide which equations are true or false. Make sure that everyone in your group agrees and can explain. 398 + 467 = 396 + 469 657 + 18 = 657 + 9 + 9 85 – 34 = 87 – 36 8 x 7 = (8 x 5) + 8 9 x 7 = (10 x 7) – 7 16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
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Learning Outcomes Students will be able to:	Explain and justify relationships between numbers in an equation. Write statements of equivalence in words and using notation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equivalent, equal sign.
Sharing back/Connect	Select student solution strategies that use relational reasoning. **Connect:** Ask students to write their own true and false number sentences.

	Notice flexibility in the use of the equals sign and whether they can develop number sentences with relationships over the equal sign. Choose some of these to use as the independent task for the second day.
Teacher Notes	● Remind students of the notation of not equal (≠) for the number sentences that they think are false. ● Students may initially treat the equals sign as an operator or indication to write the answer next. These misconceptions can be used to position students to engage in mathematical argumentation. ● Students also may compute each side to work out whether they are equal. ● Notice students who use the relationships across the equals sign to see whether there is balance. ● Highlight the students' relational responses (e.g., noticing the + 2, – 2 relationships). ● Press for use of arrows and notations to highlight the relationships. ● For the connect and independent task, have cards or strips of paper ready for students to write on and create a space called the true and false number sentence wall.
Independent Tasks	Write your own true and false number sentences. Choose some of the true and false number sentence cards to solve. Make sure you develop an explanation for why they are true and false. Give the true and false number sentences to your classmates to solve.
Anticipations	

Task 11	Can you work together in your group to solve these number sentences? Make sure that you develop an explanation of how you solved these that everyone can share. 18 + 7 = __ + 6 __ + 16 = 29 + 14 85 – __ = 86 – 28 185 – 29 = __ – 26 674 + 56 – __ = 671 73 + 5 + 3 = 73 + __
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
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Learning Outcomes Students will be able to:	Explain and justify relationships between numbers in an equation. Write statements of equivalence in words and using notation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equivalent, equal sign.
Sharing back/Connect	Select student solution strategies that use relational reasoning. **Connect:** Ask students to solve the following problems using a relational solution: __ + 219 = 177 + 218 783 – 729 = 83 – __ Support students to notice the variation in directionality between addition equivalence problems (+1, -1) and subtraction equivalence problems (-700, -700).

Teacher Notes	• Students may initially treat the equals sign as an operator or indication to write the answer next. • Students also may compute each side to work out whether they are equal. • Notice students who use the relationships across the equals sign to see whether there is balance. • Highlight to the students to look across the equals sign and find the relationships between numbers to the left and the numbers on the right. • Notice students who use the relationships across the equals sign to see whether there is balance. • Press for use of arrows and notations to highlight the relationships.
Independent Tasks	Solve these equations: 16 + 9 = __ + 8 __ + 18 = 25 + 16 63 – __ = 73 – 28 132 – 47 = __ – 45 453 + 67 – __ = 451 69 + 4 + 2 = 69 + __
Anticipations	

Task 12 (Whole class option)	Maryssa said "When you are multiplying two numbers together it doesn't matter which order you multiply them in, the product will be the same". Work in a group and explore whether you agree or disagree with this statement. Does this work for all numbers? Does it work for addition, subtraction, and division?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.
Curriculum Links	**NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-2:** Know basic multiplication and division facts. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers. **NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-8:** Generalise properties of multiplication and division with whole numbers.
Learning Outcomes Students will be able to:	Explain and justify how the commutative property of whole numbers applies to addition and multiplication.
Mathematical language	Commutative property, factors, product, conjecture, generalisation, counter-example, special cases.
Sharing back/Connect	Select students that have used multiple representations to develop concrete forms of proof related to the conjecture. Support students to explain how their model would apply to any numbers. Highlight to students that letters or symbols can be used in maths to represent any numbers. **Connect:** Can you represent the conjectures that you have made using a statement, a diagram and a number sentence using symbols or letters to represent any number?

Teacher Notes	• Students may begin by testing different examples with numbers and different types of numbers (e.g., large, small, positive, negative, fractions, decimals). After they have explored multiple examples, prompt them by asking whether they can prove it would work with **every** number. • Have appropriate equipment for students to build concrete models to prove their conjectures (e.g., counters, grid paper, peg boards). • Look for students drawing on the commutative property and understanding that it works for multiplication and addition but not for subtraction and division. • Students may generate counter examples to prove the commutative property does not apply to subtraction or division. Students may also generate special cases (e.g., 5 – 5 = 5 – 5).
Independent Tasks	Lola's teacher asks her to solve 36 ÷ 12 = Lola thinks that she can solve the problem by taking away 12. How do you think Lola would solve this? Would this always work? Can you test this with different numbers? What is a conjecture that you can make related to division and subtraction?
Anticipations	

Task 13	Tasa is working out if the number sentences are true or false 14 x 6 = (10 x 6) + (4 x 6) 32 x 3 = (30 x 3) + 2 17 x 4 = (8 x 4) + (8 x 4) 24 x 15 = (12 x 15) + (12 x 15) He notices patterns when working out which are true or false. What do you think he notices? Does this always work? Use the equipment (grid paper, counters to build arrays) to explore the relationship. Can you write your own examples using different numbers?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.
Curriculum Links	**NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-2:** Know basic multiplication and division facts. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-8:** Generalise properties of multiplication and division with whole numbers.
Learning Outcomes Students will be able to:	Explain and justify the distributive property of multiplication. Use different representations to justify.
Mathematical language	Distributive property, factors, equivalence, conjecture, generalisation.
Sharing back/Connect	Select students who have used the relationships across the equals sign to justify equivalence rather than finding the products. Select students that have used multiple representations to develop concrete forms of proof related to the conjecture. Support students to explain how their model would apply to any numbers.

	Generalise: Can you write a range of number sentences that would match the following number sentences: 6 x 14 = 25 x 17 = Look for students drawing on the distributive property and understanding that you could adjust relationally to find all options.
Teacher Notes	• Students may focus on finding the answers for each number sentence. Position them instead to recognise the relationship across the equal sign instead of calculating the products. • Have appropriate equipment for students to build concrete models to prove their conjectures (e.g., counters, grid paper, peg boards).
Independent Tasks	 Write your own set of number sentences to describe this in as many ways as possible. Make connections across the number sentences. What patterns do you notice? Why do your patterns work? Will these work with other numbers? Can you write them as a generalisation?
Anticipations	

Task 14 (Whole class option)	Is the number that goes in the __, the same number in both of these equations? 2 x __ + 15 = 31 2 x __ + 15 – 9 = 31 – 9 Explain why or why not. Would this work with other operations? Write number sentences that use the same pattern and relationship. Can you make a conjecture from this problem?
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA3-1:** Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages. **NA3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA3-7:** Generalise the properties of addition and subtraction with whole numbers **NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-8:** Generalise properties of multiplication and division with whole numbers.
Learning Outcomes Students will be able to:	Explain, and justify that the properties of equality. Generalise the properties of equality.
Mathematical language	Equality, equation, equivalence, conjecture, generalisation.
Sharing back/Connect	Select students who have used the relationships across the equal sign to justify equivalence rather than solving the equation through substituting numbers. Ask students to share the conjectures that they have developed that are related to the properties of equality.

	Connect: Can you represent the conjectures that you have made using a statement, a diagram and symbols e.g., If a x $b + c = d$ then a x $b + c – e = d – e$
Teacher Notes	• Students may focus on substituting numbers to find the answers for each number sentence. Position them instead to recognise the relationship across the equal sign instead of calculating. • Expect students to make conjectures and generalisations. • Have concrete material available (peg boards, counters) so students can build a model to provide concrete proof for their conjecture.
Independent Tasks	Select one or more of the following assessment tasks (attached at the end of the document) as the independent activity: N1B: Addition and subtraction problems to solve. N16B: Multiplication and division problems to solve. N4A: Properties of numbers and operations.
Anticipations	

DMIC

DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

NUMBER - ADD / SUB: LEVEL 3 Task N1B

Sose has 97 Pokemon cards in her collection. She wins another 48 Pokemon cards. How many Pokemon cards does Sose have altogether? Prove and justify your answer.

Brandon is playing a video game. He scores 522 points. His sister Louisa scores 385 points. How many more points did Brandon score? Prove and justify your answer.

At the athletics competition, Alexi jumped 3.35 metres for the long-jump. Sima jumped 2.8 metres. Who jumped further and by how much? Solve the problem and justify your answer.

Write one or more word problems for a friend involving addition or subtraction. Show how you would solve it.

DMIC

DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

NUMBER - MULT / DIV: LEVEL 3 Task N16B

The school is going on a trip and has ordered 17 buses. Each bus can take 28 children. How many children can go on the trip?

The library is moving. They have 484 books and can fit 22 books into each box. How many boxes will be needed for the books?

Write your own multiplication or division problems. Show how you would solve them.

DMIC

DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASKS

NUMBER PATTERNS: LEVEL 3 Task NA4

27 x 12 = 95 – 27 = 72 ÷ 6 = (27 x 6) + (27 x 6)=

36 ÷ 3 = 12 x 27 = 567 + 39 =

85 – 17 = 567 + 39 + 1 = ☐ - 1 27 x 2 x 6 =

Look at the number sentences above

- Describe what patterns you can find
- Why do your patterns work?
- Do they work with other numbers?
- Will they always work? Explain and justify your thinking