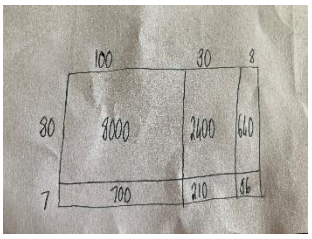
DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES

Number and Algebra

Level 4 (Year 7-8)

Teacher Booklet

Task 1	In the shopping mall carpark, there are 138 rows for car parks. Each row has spaces for 87 cars. How many cars can fit in the carpark? Can you show your solution using two different representations? In the shopping mall carpark, there are 179 rows for car parks. Each row has spaces for 76 cars. How many cars can fit in the carpark? Can you show your solution using two different representations?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-6:** Know the relative size and place value structure of positive and negative integers and decimals to three places. **NA4-8:** Generalise properties of multiplication and division with whole numbers.
Learning Outcomes **Students will be able to:**	Explain and justify the use of the distributive property in multiplication. Explain and justify the use of the associative property in multiplication. Explain and justify the use of equivalence and compensation in multiplication. Represent reasoning using different forms of notation including equations and an area model.
Mathematical language	Distributive property, associative property, area, equivalence, compensation, factor, product.
Sharing back/Connect	Select student solution strategies that have used the distributive property, associative property or equivalence and compensation. Use the correct mathematical language to describe these. *Distributive property* *138 x 87 = (100 x 80) + (100 x 7) + (30 x 80) + (30 x 7) + (8 x 80) + (8 x 7)*

	Associative property *138 x 87 = (138 x 10 x 8) + (138 x 7)* *Equivalence and compensation* *138 x 87 = (138 x 90) – (138 x 3)* Ask students to record these as equations and model representing these using the area model. If no student solves the task using the distributive property or equivalence and compensation, then introduce either solution strategy as an alternative model previously used by other students.  **Connect**: Ask students to describe how the following equation could be solved using the distributive property or equivalence and compensation: 157 x 62 =
Teacher Notes	• Prior to launching the task, ask students to calculate the following problems: 15 x 10 = 100 x 37 = 30 x 3000 = Ask the students to discuss what they notice. Be aware of students who rely on 'just add a 0' when dealing with base 10 multiplication. Explore what is happening to the numbers (getting bigger by base (10)) rather than rely on a misconception/rule. • Expect students to use equations and an area model to record solution strategies. • If students are using standard algorithm, check for procedural knowledge with understanding. The standard algorithm can be connected with the distributive property.
Independent Tasks	Solve the following equations: 147 x 78 = 153 x 36 = 546 x 49 = 209 x 67 = Explain what patterns you used to help solve the equations.

	Would the patterns always work?
Anticipations	

Task 2	In preparation for the summer markets, Monty is planting lettuce. He plants 613 lettuce plants in each row. He can fit 391 rows on their land. How many lettuces can he grow altogether? In preparation for the summer markets, Sia is planting lettuce. He plants 828 lettuce plants in each row. He can fit 421 rows on their land. How many lettuces can he grow altogether?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-6:** Know the relative size and place value structure of positive and negative integers and decimals to three places. **NA4-8:** Generalise properties of multiplication and division with whole numbers.
Learning Outcomes **Students will be able to:**	Explain and justify the use of the distributive property in multiplication. Explain and justify the use of the associative property in multiplication. Explain and justify the use of equivalence and compensation in multiplication. Represent reasoning using different forms of notation including equations and an area model.
Mathematical language	Distributive property, associative property, area, equivalence, compensation, factor, product.
Sharing back/Connect	Select student solution strategies that have used the distributive property, associative property or equivalence and compensation. **Connect:** Ask students to describe how the following equation could be solved using the distributive property: 525 x 232 =

	Use modelling to show connections between the use of the distributive property and the standard multiplication algorithm.
Teacher Notes	• Prior to launching the task, ask students to calculate the following problems: 173 x 10 = 255 x 100 = 3055 x 1000 = Ask the students to discuss what they notice. Be aware of students who rely on 'just add a 0' when dealing with base 10 multiplication. Explore what is happening to the numbers (getting bigger by base (10)) and moving one place to the right. rather than rely on a misconception/rule. • Expect students to use equations and an area model to record solution strategies. • If students are using standard algorithm procedure, check for procedural knowledge with understanding.
Independent Tasks	Solve the following equations: 836 x 261 = 319 x 672 = 467 x 789 = 876 x 208 = Represent your solution strategies using equations and the area model.
Anticipations	

Task 3	The Salvation Army has donated $3213 to help families get essential furniture to set up their homes. They have 56 families that need help to do this. How much money will each family receive? What numbers (above a thousand) could you start with, that would mean that each family only receives dollars and no cents?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-6:** Know the relative size and place value structure of positive and negative integers and decimals to three places. **NA4-8:** Generalise properties of multiplication and division with whole numbers.
Learning Outcomes **Students will be able to:**	Explain and justify the use of the partial quotients/distributive property in division. Explain and represent the inverse relationship of multiplication and division. Represent reasoning using different forms of notation.
Mathematical language	Distributive property, inverse relationship, equivalence, factor, product, quotient, divisor, dividend.
Sharing back/Connect	Select student solution strategies where they have used the inverse relationship of multiplication and division or the partial quotient/distributive property (or a mixture of both) in the solution. If either solution strategy has not been used, introduce this as a solution strategy that students have used previously. *Inverse relationship* *3213 ÷ 56 = ?* *56 x ? = 3213* *56 x 10 = 560 ….*

	Partial quotient/Distributive property *3213 ÷ 56 = (1120 ÷ 56) + (1120 ÷ 56) + (560 ÷ 56) + (112 ÷ 56) + (112÷ 56) + (112 ÷ 56) + (77 ÷ 56)* **Connect:** Ask students to describe how you would solve the following equation using either the inverse relationship or the partial quotient/distributive property: 7187 ÷ 35 =
Teacher Notes	• Prior to launching the task, ask students to calculate the following problems: 86 ÷ 21 = 866 ÷ 211 = 675 ÷ 225 = $\frac{1}{2} \div \frac{1}{4} =$ Ask students to discuss what they notice. • Notice students who are using addition or subtraction and support them to re-work as multiplicative thinking. • Notice students who use doubling and support them to recognise this as multiplying by two. Press students to use larger factors such as 5 or 10.
Independent Tasks	Solve the following equations: 5556 ÷ 25 = 8666 ÷ 422 = 7255 ÷ 35 = 9333 ÷ 322 = $\frac{1}{4} \div \frac{1}{8} =$ $\frac{1}{2} \div \frac{1}{5} =$
Anticipations	

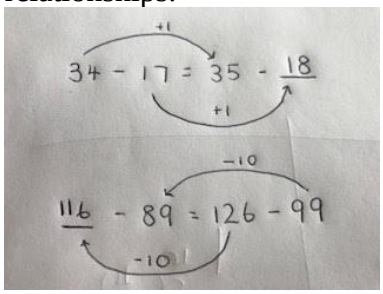
Task 4	Griffin biscuit factory use a machine to put 203 biscuit packets in a large container to be sent for packaging. Every ten minutes the machine sorts 4519 packets of biscuits. How many containers would be used every ten minutes and how many packets of biscuits would be left over? For what numbers would there be no packets of biscuits left over but almost the same number of containers used?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-6:** Know the relative size and place value structure of positive and negative integers and decimals to three places. **NA4-8:** Generalise properties of multiplication and division with whole numbers.
Learning Outcomes **Students will be able to:**	Explain and justify the use of the partial quotients/distributive property in division. Explain and represent the inverse relationship of multiplication and division. Represent reasoning using different forms of notation.
Mathematical language	Distributive property, inverse relationship, equivalence, factor, product, quotient, divisor, dividend.
Sharing back/Connect	Select student solution strategies where they have used the partial quotient/distributive property in the solution. **Connect:** Ask students to describe how you would solve the following equation using partial quotients/distributive property: 6716 ÷ 307 =

	Use modelling to show connections between the use of the partial quotients/distributive property and the standard division algorithm.
Teacher Notes	• Prior to launching the task, ask students to calculate the following problems: 77 ÷ 31 = 777 ÷ 311 = $\frac{1}{3} \div \frac{1}{6} =$ Ask students to discuss what they notice. • Notice students who are using addition or subtraction and support them to re-work as multiplicative thinking. • Notice students who use the inverse property or who are using partial quotients/distributive property in their calculations.
Independent Tasks	Solve the following equations: 7255 ÷ 35 = 9333 ÷ 322 = $\frac{1}{6} \div \frac{1}{12} =$ $\frac{1}{2} \div \frac{1}{8} =$
Anticipations	

Task 5 (Whole class option)	Work in your group to see whether you can work out the last digits of the following numbers without doing the full multiplication: 5^4 6^4 7^4 Discuss the patterns that you could use to help you with the task. Develop a range of conjectures and see whether you can prove them.
Big Ideas	Relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-8:** Generalise properties of multiplication and division with whole numbers. **NA5-2:** Use prime numbers, common factors and multiples, and powers (including square roots).
Learning Outcomes **Students will be able to:**	Identify that a power is represented by a base number and an exponent and that a power is the product of multiplying a number by itself. Calculate powers of numbers. Explain and justify patterns and relationships in powers of numbers.
Mathematical language	Power, base number, exponent, product, digit, conjecture.
Sharing back/Connect	Select and sequence student solutions which identify patterns and ask students to explain and justify these. *1, 5 and 6 are special cases of exponents as they will always end with the same digit.* *Exponents of 2, 3, 4, 7, 8, 9 each have a set pattern/cycle for the last digit.* **Connect:** Can you work out what the last digit would be for 7^5 without doing the full multiplication? Through discussion draw out that in last digit patterns, some numbers are special and only have the same number they were multiplied by, some have 2 numbers, and the others have a combination of 4 numbers. Notice students using conjecture or proof that 5 x 5 always ends with a 5 so therefore it continues always with a 5 in the ones digit.

Teacher Notes	• Prior to launching the task, ask students to work with a partner and record equations for square numbers. $1^2 = 1 \times 1 = 1$ $2^2 = 2 \times 2 = 4$ …. Ask the students to identify the patterns that they notice. Provide students with access to squared paper to draw representations. • Provide students with access to calculators to help them work on the task. • Encourage students to use smaller numbers or draw models to prove that their conjectures work.
Independent Tasks	Work to see whether you can work out the last digits of the following numbers without doing the full multiplication: 2^8 8^4 9^6 10^5 What patterns can you use that will help you with this task? Predict the results and write these down with a justification. Now use a calculator to check whether you were correct. What conjectures can you make from this?
Anticipations	

Task 6 (Whole class option)	Can you work together in your group to work out whether these number sentences are true or false? Make sure that you develop an explanation that everyone can share. 398 + 467 = 396 + 469 657 + 18 = 657 + 9 + 8 82 – 34 = 84 – 36 465 = 465 8 x 7 = (8 x 5) + 8 9 x 7 = (10 x 7) – 7 25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-8:** Generalise properties of multiplication and division with whole numbers. **NA4-7:** Form and solve simple linear equations.
Learning Outcomes **Students will be able to:**	Explain and justify relationships between numbers in an equation. Write statements of equivalence in words and using notation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equivalent, equal sign.
Sharing back/Connect	Select student solution strategies that use relational reasoning. *82 – 34 = 84 – 36 is true because 84 is two more than 82 and 36 is two more than 34.* **Connect:** Can you work out whether the following are true or false without calculating each side? 353 – 328 = 53 – 28 227 + 378 = 217 + 398
Teacher Notes	• Ensure that students understand what true and false means. Introduce notation of not equal (≠) for the number sentences

	that they think are false. • Students may initially treat the equals sign as an operator or indication to write the answer next. These misconceptions can be used to position students to engage in mathematical argumentation. • Students also may compute each side to work out whether they are equal. • Notice students who use the relationships across the equals sign to see whether there is balance. • Highlight the students' relational responses (e.g., noticing the + 2, – 2 relationships). • Press for use of arrows and notations to highlight the relationships. 
Independent Tasks	Work out which number sentences are true or false and explain your reasoning. 369 + 496 = 367 + 494 267 + 7 + 9 = 267 + 16 71 – 57 = 73 – 59 459 = 455 6 x 7 = (6 x 5) + 7 + 7 13 x 8 = (13 x 5) + (13 x 2) 4 + 5 + 6 + 7 = 8 + 9 + 10
Anticipations	

Task 7	Can you work together in your group to solve these number sentences? Make sure that you develop an explanation and justification. 189 + 25 = __ + 26 85 – __ = 75 – 28 674 + 56 – __ = 671 24 x 16 = 48 x __ 105 ÷ 15 = (45 ÷ 15) + (__÷ 15)
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-8:** Generalise properties of multiplication and division with whole numbers. **NA4-7:** Form and solve simple linear equations.
Learning Outcomes **Students will be able to:**	Explain and justify relationships between numbers in an equation. Write statements of equivalence in words and using notation. Solve equivalence problems and explain and justify the solutions.
Mathematical language	Equivalent, equal sign.
Sharing back/Connect	Select student solution strategies that use relational reasoning. **Connect:** Ask students to generate conjectures related to the equivalence problems that build on the properties of equality.
Teacher Notes	• Students may initially treat the equals sign as an operator or indication to write the answer next. • Students also may compute each side to work out whether they are equal. • Notice students who use the relationships across the equals sign to see whether there is balance. • Highlight to the students to look across the equals sign and find the relationships between numbers to the left and the numbers on the right. • Notice students who use the relationships across the equals sign to see whether there is balance.

	• Highlight the students relational responses (e.g., noticing the + 2 – 2 relationships). • Press for use of arrows and notations to highlight the relationships.
Independent Tasks	Find the missing numbers: 37 + 26 = 35 + ____ ___ + 276 = 399 +286 376 – 159 = 276 – __ = __ – 266 = 571 – 268 3 x 18 = (3 x 6) + (3 x __) 176 ÷ 8 = (__ ÷ 8) + (16 ÷ 8)
Anticipations	

Task 8 (Whole class option)	Soane is solving a division problem that his teacher gave him. He is solving this: 216 ÷ 12 = Soane solves it by writing 216 ÷ 12 = 216 ÷ 2 ÷ 3 ÷ 2 Do you agree with Soane's solution? In your group, develop an explanation of why this works or why you think it doesn't work. Can you develop examples with other numbers which also use this pattern? Does this pattern work with multiplication?
Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship. Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Patterns and relationships can be used, represented, and generalised in a variety of ways.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-8:** Generalise properties of multiplication and division with whole numbers. **NA4-7:** Form and solve simple linear equations. **NA5-2:** Use prime numbers, common factors and multiples, and powers (including square roots).
Learning Outcomes **Students will be able to:**	Explain and justify how you can solve division problems by splitting the divisor into factors. Represent that the equals sign as a statement of balance and show which operations to both sides of an equation preserve that balance.
Mathematical language	Factors, divisor, dividend, associative property.
Sharing back/Connect	Select students who use the relationship and properties rather than calculating. Highlight to the students that you do not need to calculate but can use the relationship to solve different equations. **Connect:** Elise thought that Soane should have solved the problem like this:

	216 ÷ 12 = (216 ÷ 2) + (216 ÷ 3) + (216 ÷ 2) Do you agree with Elise? Which way works and why? How would Soane solve the following problem 438 ÷ 16?
Teacher Notes	• Facilitate students to focus on the solution strategy and generalisation rather than calculating the solution. • Notice and highlight the conjectures that students develop.
Independent Tasks	Find the missing numbers: 46 x __ = 46 x 3 x 5 x 2 1392 ÷ 3 ÷ 2 ÷ 2 ÷ __ = 1392 ÷ 24 1260 ÷ __ = 1260 ÷ 2 ÷ 5 ÷ 3 ÷ 2 27 x 36 = 27 x 3 x __ x 4
Anticipations	

Task 9 (Whole class option)	Tina works at a clothing factory making t-shirts. At the factory she gets paid a specific amount per day and $5 for each t-shirt she makes. Can you write an equation that you could use to work out how much she earns in a day for making any number of t-shirts? Tina earned $50 for one day. Can you show the different ways that Tina could have earned this using your equation?
Big Ideas	Mathematical situations can be represented using variables, expressions, and equations.
Curriculum Links	**NA4-7:** Form and solve simple linear equations. **NA5-4:** Use rates and ratios.
Learning Outcomes **Students will be able to:**	Form an equation to represent a situation in a problem. Explain and justify how when the value of one variable is known the value of the other variable can be found by solving the equation. Represent variables as any letter of choice.
Mathematical language	Variable, unknown, equation.
Sharing back/Connect	Select student solution strategies that have represented the unknown using a variable. **Connect:** $2n - 5 =$ Can you write a story/context that matches the equation?
Teacher Notes	• Before you launch the problem, ask the students to work with a partner and write an expression to match these situations: a) I have some lollies and I get five more. b) I have some lollies and I get five more and then I get three more. c) I have some lollies and I give six away. c) I have some lollies and I get five more and then I double the total amount of lollies I have. Introduce each one and share student solutions that use a variable to model the equation. If all students put numbers, then problematise this by asking, do we know how many? If we don't know, we can use a letter to represent any number. • Introduce to students that $2n = 2 \times n$ • Terminology such as unknown variable will need to be explored. Avoid students using *t* to represent t-shirts as this can lead to a letter as an object misconception related to variables.

Independent Tasks	Nikki and Milo each have a sticker collection. They know that their sticker collections each contain the same number of stickers, but they don't know how many. Milo also has 12 stickers on a sheet. How would you represent the number of stickers that Nikki has? How would you represent the total number of stickers that Milo has? Nikki and Milo combine all their stickers to make one collection. How would you represent the total number of stickers they have? Write other stories with unknowns and show how these would be represented using equations and expressions.
Anticipations	

Task 10	Work together in your group to solve these equations and justify your solution. Make sure that everyone can explain and justify your responses. $w + 14 = 30$ $2b + 5 = 23$ $6h - 7 = 29$ $d + d - 5 = 13$ $3p + p + 2 - p = 17$
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. A solution to an equation is a value of the unknown or unknowns that makes the equation true. Properties of equality and the inverse property can be used to generate equivalent equations and find solutions.
Curriculum Links	**NA4-7:** Form and solve simple linear equations.
Learning Outcomes **Students will be able to:**	Explain and justify how when the value of one variable is known the value of the other variable can be found by solving the equation. Use inverse relationships and understanding of properties of equality to solve equations.
Mathematical language	Unknown, variable, inverse relationships, equivalence, equation, values.
Sharing back/Connect	Select student solution strategies that use inverse relationships and the properties of equality. **Connect:** The solution to $2n + 15 = 31$ is $n = 8$. What is the solution to $2n + 15 - 9 = 31 - 9$? What conjecture can you make from this? Conjecture could be represented as: If $a + b = c$ then $a + b - d = c - d$ or

	If $a + b = c$ then $a + b + d = c + d$
Teacher Notes	• Before you launch the task, ask the students to work with a partner and solve: What are the possible values for $k + k = 12$? What are possible values for $j + s = 10$? Highlight that k will be the same number so only one solution. Address potential misconception that j and s cannot both equal 5. Highlight that j + s has multiple solutions including j = 5 s = 5 • Values of variables: Variables can have any values - a letter is assigned to this value. The same variable has the same value in an equation. Different variables can have the same value. • Discuss and explore with the students that equations remain balanced as long as you use the properties of equality. An operation conducted on one side of the equal sign must be applied on the other. Apply the inverse of an operation to cancel it out or remove it. The goal in solving linear equations is to isolate the unknown variable by applying the inverse to remove other known variables.
Independent Tasks	Solve the following equations: $5g = 35$ $d + 7 = 16$ $k - 9 = 31$ $n \div 4 = 3$ $3b + 4 = 28$ $10x - 14 = 26$ $8j + 7 = 39$ $3e - 8 = 28$
Anticipations	

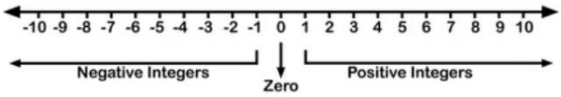
Task 11	Work together in your group to solve these equations and justify your solution. Make sure that everyone can explain and justify your responses. $6q = 2q + 24$ $2s + 5s = 15 + 13$ $16 = 4 – t + 3t$ $15 + p = 2p – 3$ $7y – 13 = 2y + 12$
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. A solution to an equation is a value of the unknown or unknowns that makes the equation true. Properties of equality and the inverse property can be used to generate equivalent equations and find solutions.
Curriculum Links	**NA4-7:** Form and solve simple linear equations.
Learning Outcomes **Students will be able to:**	Explain and justify how when the value of one variable is known the value of the other variable can be found by solving the equation. Use inverse relationships and understanding of properties of equality to solve equations.
Mathematical language	Unknown, variable, inverse relationships, equivalence, equation, values.
Sharing back/Connect	Select student solution strategies that use inverse relationships and the properties of equality to isolate and solve for the unknown. **Connect:** Ask students to describe the steps that you could take to solve the following: $4a + 6 = 9a – 9$
Teacher Notes	• Before you launch the task, revisit the conjectures made in the previous lesson that established that equations remain balanced as long as you do the same thing to both sides. Ask the students: What value would make the following equation true? $j + j = j + 6$

	Is $b + f + n = b + e + n$ always, sometimes, or never true? Highlight that the same variable has the same value in an equation but different variables can have the same value. • Discuss and explore with the students that equations will remain balanced as long as you use the properties of equality. An operation conducted on one side of the equal sign must be applied on the other. Apply the inverse of an operation to cancel it out or remove it. The goal in solving linear equations is to isolate the unknown variable by applying the inverse to remove other known variables.
Independent Tasks	Solve the following equations: $x + 11 = 40$ $23 - h = 15$ $8e = 80$ $6h - 5 = 7$ $1 + 2r = 35$ $8q + 8 = 2q + 62$ $3h + 4 = h + 16$ $6w - 8 = 13 + 3w$
Anticipations	

Task 12	In your groups look at the equations and develop a story that matches the equation. Make sure that everyone in your group can explain and justify why the story matches the equation. Have a go at solving the story problems that you have created: 5 + __ = 21 -5 – 3 = __ – -15 = -7
Big Ideas	Mathematical situations can be represented as equations which include both positive and negative integers. A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-2:** Understand addition and subtraction of fractions, decimals and integers
Learning Outcomes **Students will be able to:**	Represent in a mathematical story a situation in a problem which has both negative and positive numbers. Explain and justify how integers are an extension of whole numbers and include both positive and negative whole numbers.
Mathematical language	Integers, negative number, positive number.
Sharing back/Connect	Select student solutions that can be compared for key similarities and differences. **Connect:** Ask students to compare the stories and notice similarities and differences.
Teacher Notes	• Before you launch the task, ask students to brainstorm everything that they know about negative numbers and record their ideas. • Integers are an extension of whole numbers which include positive and negative whole numbers that are opposites (...-2, -1, 0, 1, 2,...). • Possible story contexts: height above sea level, scoring in sports/games e.g. golf, Bridge, positive/negative spaces e.g. digging holes, lifts which go below ground, temperatures below zero.
Independent Tasks	Look at the equations and develop one or more stories that match each equation.

	4 + -5 = - 13 – __ = 0 __ + 32 = - 31 22 – -14 =
Anticipations	

Task 13	In your groups represent your reasoning on a number line to show how you solved each of these problems: -3 + 4 = 12 – 17 = -4 + -4 = 2 + -5 = 3 – -5 = -9 + -8 =
Big Ideas	Mathematical situations can be represented as equations which include both positive and negative integers. A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-2:** Understand addition and subtraction of fractions, decimals and integers. **NA4-6:** Know the relative size and place value structure of positive and negative integers and decimals to three places.
Learning Outcomes **Students will be able to:**	Solve simple addition and subtraction equations using integers. Use a number line to represent the relationship between positive and negative integers in equations. Explain and justify the role of zero as neither positive nor negative. Explain and justify the use of - as an operation symbol (subtraction) and direction symbol (direction/size of movement) for negative numbers.
Mathematical language	Integers, negative number, positive number.
Sharing back/Connect	Select student solution strategies that will support a discussion of the relationship between addition and subtraction when working with integers. **Connect:** What patterns did you notice when you were adding and subtracting positive and negative numbers?
Teacher Notes	• Before you launch the task, ask students to share when they see negative numbers in life? [Lifts, mortgages, temperature] • Model an empty number line on the board and ask students to discuss where the numbers would go if you were counting from negative 5 to positive 5. Use the number line to

	represent the location/relationship of negative/positive numbers to each other.  • Highlight that two integers that are the same distance from the origin in opposite directions are called opposites and when added cancel each other making 0. • Highlight difference between the use of - as an operation symbol (subtraction) and direction symbol (direction/size of movement) for negative numbers. • Consider using physical materials to represent positive and negative numbers e.g. black counters (positive) and red counters (negative).
Independent Tasks	Solve these equations (use an empty line if it helps): -18 + 5 = - 23 + - 18 = -37 – -42 = 63 – -24 = -143 – 69= -145 + -251 = 274 – -128 =
Anticipations	

Task 14	Can you work together in your group to work out whether these number sentences are true or false? Make sure that you develop an explanation that everyone can share. 8 + 3 = -8 – 3 7 + 5 = 7 + -5 -3 + 6 = 6 + -3 10 – 4 = 10 – -4 -7 + - 9 = -7 – -9 - 5 – -5 = -5 – -1 In your group, talk about the patterns that you notice and be ready to share these.
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other. Mathematical situations can be represented as equations which include both positive and negative integers. A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.
Curriculum Links	**NA4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA4-2:** Understand addition and subtraction of fractions, decimals and integers **NA4-6:** Know the relative size and place value structure of positive and negative integers and decimals to three places
Learning Outcomes **Students will be able to:**	Solve simple addition and subtraction equations using integers. Use a number line to represent the relationship between positive and negative integers in equations. Explain, justify and represent reasoning related to maintaining equality between operations which involve integers.
Mathematical language	Integers, negative number, positive number.
Sharing back/Connect	Select student solution strategies that use the properties of equality and understanding of negative numbers. **Connect:** What conjectures can you make that will always work about adding and subtracting negative and positive numbers?

Teacher Notes	• Notice student solution strategies that use the properties of equality. • Highlight the difference between the use of - as an operation symbol (subtraction) and direction symbol (direction/size of movement) for negative numbers.
Assessment Tasks	Select one or more of the following assessment tasks (attached at the end of the document) as the independent activity: N45: Multiplication and division problems to solve. NA5: Properties of numbers and operations. N4: Integers and negative numbers.
Anticipations	

DMIC

DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

NUMBER - MULT / DIV: LEVEL 4 Task N45

At the event centre, there are 225 rows of seats. Each row has 179 seats in it. How many seats are there altogether?

The library is moving. They have 3248 books and can fit 76 books into each box. How many boxes will be needed for the books?

Write your own multiplication or division problems. Show how you would solve them.

DMIC

DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASKS

NUMBER & ALGEBRA: LEVEL 4 Task N5A

76 x 15 = 37 + 43 + 40 + 36 = 99 ÷ 3 ÷ 3 =

7 x 86 = 99 ÷ 9 = 4 x 66 =

6^3= (70 x 5) + (70 x 10) + (6 x 10) + (6 x 5) = 12 x 22 =

37 + 40 + 36 + 43 = 6 x 6 x 6 = (7 x 90) - (7 x 4) =

Look at the number sentences above.

- Describe what patterns you can find.
- Why do your patterns work?
- Do they work with other numbers?
- Will they always work? Explain and justify your thinking

DMIC
DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

NUMBER – INTEGERS: LEVEL 4 Task N4

Why do we need negative numbers?

Give examples of how negative numbers can be useful.