

MASSEY UNIVERSITY
Institute of Fundamental Sciences
Mathematics

160.734 Studies in Applied Differential Equations

Semester Test

Semester One — May 2017

Time allowed: **55 minutes**

This is a **closed book** examination.

Total marks: 40

Attempt all questions. There are 6 questions altogether.
Be sure to read each question carefully.

Show all working for full credit.

1. Compute e^{tA} where $A = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$. HINT: A is nilpotent.

[5 marks]

2. Consider the linear system

$$\begin{aligned}\dot{x} &= \alpha x - y, \\ \dot{y} &= (\alpha + 1)x.\end{aligned}$$

Determine the values of $\alpha \in \mathbb{R}$ for which the equilibrium $(x, y) = (0, 0)$ is Lyapunov stable but not asymptotically stable.

[6 marks]

3. Consider the system

$$\begin{aligned}\dot{x} &= (x+1)^2 - y, \\ \dot{y} &= x - y + 3.\end{aligned}$$

(a) Find all equilibria of the system.

(b) Classify each equilibrium as a stable node, a stable focus, an unstable node, an unstable focus, a saddle, or none of the above.

[3 + 6 = 9 marks]

4. Suppose that $\mathbf{x}^*$ is an equilibrium of a four-dimensional system $\dot{\mathbf{x}} = f(\mathbf{x})$ with

$$Df(\mathbf{x}^*) = \begin{bmatrix} -1 & 1 & 1 & -2 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & 4 & 3 \end{bmatrix}.$$

What are the dimensions of $W^s(\mathbf{x}^*)$ and $W^u(\mathbf{x}^*)$? Provide at least one sentence of explanation.

[4 marks]

5. Consider the system

$$\begin{aligned}\dot{x} &= y + y^2, \\ \dot{y} &= 2x - y + 3y^2.\end{aligned}$$

For what values of a and b is the stable manifold of the origin $W^s(0,0)$ given by

$$y = ax + bx^2 + \mathcal{O}(x^3) ?$$

[10 marks]

6. Determine the value of μ at which

$$\dot{x} = \mu + \frac{1}{2}x - x^4,$$

has a saddle-node bifurcation.

[6 marks]

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