# Abstracts

# Approximation assisted estimation of eigen vectors under quadratic loss

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The estimation of eigen vectors of covariance matrix is considered in the presence of prior information regarding the parameter vector of interest. This information in practice will be available in any realistic problem. Like statistical models underlying the statistical inferences to be made, the prior information will be susceptible to uncertainty and the practitioners may be reluctant to impose the additional information in the estimation process. However, a very large gain in precision may be achieved by judiciously exploiting the information. In this talk, I will discuss how to combine sample and non-sample information. The estimators based on the shrinkage and other rules are proposed. The expressions for the risks under quadratic loss of the estimators are derived and compared with the benchmark estimator. For illustration purposes, the method is applied to real datasets.

# Convergence properties of alternating Markov chains

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Suppose we have two Markov chains defined on the same state space. What happens if we alternate them? If they both converge to the same stationary distribution, will the chain obtained by alternating them also converge? Consideration of these questions is motivated by the possible use of two different updating schemes for MCMC estimation, when much faster convergence can be achieved by alternating both schemes than by using either singly (Jones, 2004).

**References**

Jones G. (2004). Markov chain Monte Carlo Estimation for the Two-Component Model, *Technometrics*, **46**, pp99-107.

# Asymptotic birth-death processes: A matrix analysis approach

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A denumerable Markov Process with the state space $Z_+$ is called an asymptotic birth-death process if there is a finite subset G of $Z_+$ for which the restriction of the infinitesimal generator of the process, the so called q-matrix Q, to $Z_+ \backslash G$ is a birth-death q-matrix. Matrix analysis approach is applied to such structure. Using this approach, the close relationship between asymptotic birth-death processes and the well-developed birth-death processes is revealed. Regularity criteria for such process are established. Properties of such structure are investigated. In particular, conditions for recurrence and positive recurrence are obtained. Equilibrium distributions are given for the ergodic case. Some useful information regarding the transient asymptotic birth-death processes is also provided. The probabilistic interpretations of the results are explained.

# Modeling multivariate meta-analysis using bootstrap resampling techniques

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Meta-analysis is a powerful analytical tool in research synthesis for effective evidence based decision making. The end points of several studies are pooled together in a systematic way to derive an overall statistic. The extended multivariate meta-analysis involves the synthesis of studies that may report several related variables. The outcomes, the predictors or both the outcomes and the predictors may be multivariate in nature. In such situations, the variance-covariance matrix structure needs to be taken into account for effective pooling of the data. The bootstrap techniques are distribution free, computer intensive and they work very well with minimal assumptions. In bootstrap approach, the studies are first replicated using appropriate weights. This collection of replicates is considered as the bootstrap population. From this population, repeated samples are taken and for every sample the statistics of interest are computed. This innovative meta-analytical method not only gives better insight to the area of research but also simplifies the mathematical complexity. In this paper we discuss the advantages of bootstrap resampling techniques in multivariate meta-analysis.

# Decomposing the Watson efficiency in partitioned linear models

Ka Lok Chu[1], Jarkko Isotalo[2], Simo Puntanen[2]
and George P.H. Styan[3]

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While considering the estimation of regression coefficients in a partitioned weakly singular linear model, Chu, Isotalo, Puntanen & Styan (2004; in press) introduced a particular decomposition for the Watson efficiency of the ordinary least squares estimator. This decomposition presents the "total" Watson efficiency as a product of three factors. In this paper we give new insight into the decomposition showing that all three factors are related to the efficiencies of particular sub-models or their transformed versions. Moreover, we prove an interesting connection between a particular split of the Watson efficiency and the concept of linear sufficiency. We shortly review the relation between the efficiency and specific canonical correlations. We also introduce the corresponding decomposition for the Bloom-field-Watson commutator criterion, and give a necessary and sufficient condition for its specific split.

# Continuous canonical correlation analysis

C. M. Cuadras

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We obtain the eigenvalues and eigenfunctions of a covariance kernel with respect to another kernel, related to the Cuadras-Auge copula, the survival copula for the Marshall-Olkin distribution. Then we obtain the set of canonical correlations and functions for this copula and prove that they have continuous dimensionality. The maximum correlation is the dependence parameter and the canonical function is the Heaviside distribution.

# Partially diffuse starting values in state space models

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I will discuss when two specifications of partially diffuse starting values give the same results in all state space models. I will also discuss some of the difficulties in giving a clean and simple justification of the results.

# The geometry of statistical efficiency

K. Gustafson

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We will place certain parts of the theory of statistical efficiency into the author's operator trigonometry, thereby providing new geometrical understanding of statistical efficiency. Important earlier results of Bloomfield and Watson, Durbin and Kendall, Rao and Rao, will be so interpreted.

# What are *the* residuals for the linear model?

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We consider residuals for the linear model with general covariance structure. In contrast to the situation where observations are independent, there is no unique answer to the question posed in the title. We propose a taxonomy and draw attention to three quite distinct types of residual: the marginal residual, the conditional residual and the innovation residual. We adopt a very broad perspective including miixed models, time series, and smoothers as well as models for spatial and multivariate data. We concentrate on defining the three different types of residual and discusing their interelationships.

# On singularly perturbed Markov chains

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Nothing structurally happens in case that transition probabilities in an ergodic Markov chain are slightly perturbed as all measures are continuous with the perturbation. This is not the case when the state space can be decomposed into a number of ergodic classes plus a number of transient states, and when the perturbation coupled them all. For example, the stationary distribution is now uniquely defined and mean passage times between states belonging to different classes are now well defined. The talk will survey some results which are old and some which are under construction. Special emphasis will be given to the issue of time scales.

# Simple procedures for finding mean first passage times in Markov chains

Jeffrey J. Hunter

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The derivation of mean first passage times in Markov chains involves the solution of a family of linear equations. By exploring the solution of a related set of equations, using suitable generalized inverses of the Markovian kernel I - P, where P is the transition matrix of a finite irreducible Markov chain, we are able to derive elegant new results for finding the mean first passage times. As a by-product we derive the stationary distribution of the Markov chain without the necessity of any further computational procedures. Standard techniques in the literature, using for example Kemeny and Snell's fundamental matrix Z, require the initial derivation of the stationary distribution followed by the computation of Z, the inverse of $I - P + \mathbf{e}\pi^T$ where $\mathbf{e}^T = (1, 1, \ldots ,1)$ and $\pi^T$ is the stationary probability vector. The procedures of this paper involve only the derivation of the inverse of a matrix of simple structure, based upon known characteristics of the Markov chain together with simple elementary vectors. No prior computations are required. Various possible families of matrices are explored leading to different related procedures. Applications to perturbed Markov chains are discussed.

# Hessian equivalence to bordered Hessian

Eric Iksoon Im

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The second-order sufficient conditions are traditionally given in terms of bordered Hessian matrix for constrained optimization problems, whereas they are given in terms of Hessian matrix for unconstrained problems. This dichotomy appears to bother some practitioners of optimization. In this paper, we show that the second order conditions can be stated solely in terms of Hessian matrix for both constrained and unconstrained cases, thus providing a unified criterion for testing the second–order conditions.

# Comparison of the ordinary least squares predictor and the best linear unbiased predictor in the general Gauss-Markov model

J. Isotalo and S. Puntanen

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In this paper we consider the prediction of new observations in a general Gauss-Markov model. We compare properties two alternative predictors: the best linear unbiased predictor, BLUP, and the ordinary least squares predictor, OLSP. In particular, we focus on questions related to equality of two predictors BLUP and OLSP, and to the efficiency of the ordinary least squares predictor with respect to the best linear unbiased predictor.

# Inverse of the Information Matrix

J. A. John

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When perturbing a regression model, to identify outliers and influential observations, most computer packages use updating procedures to find the inverse of the information matrix. Consider a linear model partitioned into two components, one of which is perturbed and the other left unchanged. In the talk we shall examine how the updating procedure can be extended to the inverse of the information matrix of the perturbed component. We shall also briefly discuss a three component model where the third component changes as the first component is perturbed. The results have important applications in the construction of efficient experimental designs.

# Covariance decomposition for Gaussian graphical models

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This talk will examine a new strategy for interpreting Gaussian graphical models, one which potentially will provide new knowledge about how variables interact. This strategy is based on decomposition of the covariance between two variables into a sum of path weights for all paths connecting the two variables in an undirected graph. Extensions to graphs initially inferred as acyclic directed graphs are also considered. This decomposition is derived using basic identities from linear algebra, and is feasible for very large numbers of variables if the corresponding precision matrix is parse. The resulting path weights can be used to highlight the most important intermediaries between two correlated variables. The talk will be illustrated with an example using gene expression data.

# Some properties of transition matrices for chain binomial models

G. Jones

*Institute of Information Sciences and Technology, Massey University, New Zealand*

A chain binomial model is a Markov chain with a transition matrix whose rows are binomial probabilities. Two such chains will be presented and illustrated with possible applications. The talk will focus in particular on some interesting properties of the transition matrices.

# Innovations, incomplete Fisher information matrices and empirical processes

Estate V. Khmaladze

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Suppose $\Xi = (X_1, \ldots, X_n)$ is an $n$-dimensional vector of independent random variables, each with normal distribution $N(0,1)$. Its projection $\hat{\mathrm{X}} = (X_1 - \bar{X}, \ldots, X_n - \bar{X})$, where $\bar{X} = \sum_{i=1}^{n} X_i / n$, into the subspace orthogonal to $\mathrm{I} = (1,1\ldots,1)$ has coordinates which are dependent random variables. However, let us couple $\hat{\mathrm{X}}$ with the filtration $\{\Phi_k\}_{k=1}^{n}$ where each σ-algebra is generated by its first $k$ coordinates:

$$\mathrm{F}_k = \sigma\{X_1 - \bar{X}, \ldots, X_k - \bar{X}\}, \qquad k = 1, \ldots, n,$$

and let us consider the so called *innovations* for $(\hat{X}, \{F_k\}_{k=1}^{n})$:

$$Y_k = X_k - \bar{X} - E[X_k - \bar{X} \mid \mathrm{F}_{k-1}] = X_k - \frac{1}{n-k+1} \sum_{i=k}^{n} X_i$$

This conditioning on the "past" gives us the vector $\Psi = (Y_1, Y_2 \ldots, Y_n)$ again with independent coordinates. At the same time $\Psi$ is in one-to-one correspondence with $\hat{\mathrm{X}}$. So, it carries the same statistical "information", but has a simpler distribution than $\hat{\mathrm{X}}$.

This construction in continuous time framework of empirical processes leads to the following substantial gain. Let $P_n(\cdot)$ be empirical distribution function of some sample and $P_n(\cdot, \theta)$ be some hypothetical distribution function, depending on the parameter $\theta$, which has to be estimated from the sample. The *estimated* empirical process

$$\hat{\upsilon}_n(x) = \sqrt{n}(P_n(x) - P(x, \hat{\theta}))$$

converges in distribution to projection $\hat{\upsilon}$ of Brownian motion (see, e.g., (Khmaladze, 1979) (Khmaladze, Koul, 2004)). It is not convenient to use functionals $\psi(\hat{\upsilon}_n)$ as test statistics because the distribution of $\psi(\hat{\upsilon})$ is not easy to find. However, by considering

$$\mathrm{F}_x = \sigma\{\hat{\upsilon}(y), y \le x\}$$

we can instead construct the innovation process $\omega(\cdot)$ for the process $\{\hat{\upsilon}(x), F_x, -\infty < x < \infty\}$ which will be simply a Brownian motion and, at the same time, will be in one-to-one correspondence with $\hat{\upsilon}(\cdot)$. Thus, empirical version $\omega_n(\cdot)$ of $\omega(\cdot)$ will preserve all statistical "information" which lies in $\hat{\upsilon}_n(\cdot)$ and the distribution of the functionals $\psi(\omega(\cdot))$ will be relatively easy to find.

In the construction of the innovation process $\omega_n(\cdot)$ the inverse of *incomplete extended Fisher information matrix* $C(\cdot,\theta)$ plays a central role. Namely, if $p(x,\theta)$ is a pdf of $P(x,\theta)$ consider the extended score function

$$q(x,\theta)^T = [1, \frac{\partial \ln p(x,\theta)}{\partial\theta}].$$

Using it construct the matrix

$$C(z,\theta) = \int_z^\infty q(x,\theta)q(x,\theta)^T p(x,\theta)dx.$$

Then the innovation process $\omega_n(\cdot)$ is

$$\omega_n(x) = \sqrt{n}\left[P_n(x) - \int_{-\infty}^x q(z,\theta)^T C^{-1}(z,\theta)\int_z^\infty q(y,\theta)P_n(dy)P(dz,\theta)\right].$$

For some families the matrix $C(z,\theta)$ is degenerate for some $z$ but one can prove that $\omega_n$ does not depend on the choice of the generalized inverse (see, e.g., Zigroshvili(1998)).

To the best of the author's knowledge, the question of serious applications of this approach to linear models remains open.

## References


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Khmaladze, E.V., Koul, H.L. (2004). Martingale Transforms Goodness-of-Fit Tests in Regression Models. *Ann.Statist.*, **32**, N3, 995-1034.

Zigroshvili, Z.P. (1998). Generalised inverse and martingale transform for empirical processes. *Georgian Mathemat. J.*, **6**, No.4.

# Approximation of the parameter distributions of growth curve model

T. Kollo [1] and D. von Rosen [2]

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We are going to approximate density functions of the maximum likelihood estimators of the parameter matrices $B$ and $\Sigma$ in the Growth Curve Model:

$$X = ABC + \Sigma^{1/2} E$$

with known matrices $A$ and $C$. In both cases we rely on a general multivariate density expansion introduced in Kollo, von Rosen (1998). The main interest is focused to the parameter $B$. When approximating density of the parameter matrix $B$ a mixture of two matrix distributions is obtained. One of the distributions is normal and the other one an elliptical distribution from the class of Kotz-type distributions (Fang, Kotz, Ng, 1990, p. 69). The first term in the remainder of the density expansion is of order $O(n^{-2})$ what refers to a good approximation.

We are concentrating to the mixture of the two distributions which appear in the expansion. The basic characteristics are examined for both, the Kotz type distribution and the mixture of interest. The program of the workshop will be of interest to anyone concerned with matrices, statistics, data analysis, and computational issues. For references, use the style of the examples below. The references should be in alphabetical order.

## References

Fang, K.-T., Kotz, S., Ng, K.-W. (1990). *Symmetric Multivariate and Related Distributions*, Chapman and Hall, London.

Kollo, T., von Rosen, D. (1998). A unified approach to the approximation of multivariate densities. *Scand. J. Statist.*, pp93-109.

# Invariant estimator in a quadratic measurement error model

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An adjusted least squares (ALS) estimator is derived that yields a similarity invariant and consistent estimate of the parameters of multivariate implicit quadratic measurement error model (IQMEM). Consistency means that the estimate converges to the true value of the parameter, as the sample size tends to infinity. In addition, a consistent estimator for the measurement error noise variance is proposed. Important assumptions are: (1) all errors are i.i.d. and (2) the error distribution is rotation-invariant. The estimators for the quadratic measurement error model are used to estimate consistently conic sections and ellipsoids.

In the IQMEM, the ordinary least squares (OLS) estimator is inconsistent, and due to the non-linearity of the model, the orthogonal regression (OR) estimator is inconsistent as well. Simulation examples, comparing the ALS estimator with the OLS method and the OR method, are discussed for the ellipsoid fitting problem.

The results are joint with Prof. S.Van Huffel and Dr. I.Markovsky (Belgium), and Dr. S.Shklyar (Ukraine). The consistency is shown in Kukush et al. (2004), and the numerical algorithm is proposed in Markovsky et al. (2004).

## References

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