## DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES

Number: Decimals and Percentages Level 4 (Year 7 - 8) Teacher Booklet

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Task 1	What percentage of your one whole container is filled with water? Be ready to explain and justify how you know.
	What percentage have you downloaded of that app? How much more would you need to download to complete it?
	Record using a range of different representations including symbols and be ready to explain and justify how they are equivalent.
Big ideas	A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line. Percent is relative to the size of the whole. A percent is a special type of ratio where a part is compared to a whole and the whole is 100. A decimal is another name for a fraction and thus can be
Curriculum links	associated with the corresponding point on the number line. NA3-1: Use a range of additive and simple multiplicative
	<ul> <li>strategies with whole numbers, fractions, decimals, and percentages.</li> <li>NA3-4: Know how many tenths, tens, hundreds, and thousands are in whole numbers.</li> <li>NA3-5: Know fractions and percentages in everyday use.</li> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals.</li> <li>NA4-5: Know the equivalent decimal and percentage forms for everyday fractions.</li> </ul>
Learning Outcomes: Students will be able to:	<ul> <li>Explain and justify the comparison of a part to the whole.</li> <li>Represent reasoning using different forms of notation, including words.</li> </ul>
Mathematical language	Percent, percentage, whole, fraction, fractional number, decimal number, rational number, equal, equivalent
Sharing back/Connect	Select students to share after each estimation in the first one who are able to explain and justify the numerical value that they have agreed on using their fingers to show numerical splitting and fraction or percent terms. In the second task, select students who can explain and justify the numerical value and have represented their estimation in multiple ways.
	Connect:
	What is the fraction, percentage, or decimal equivalences for: 25%, $\frac{1}{2}$ , 33%, .99, .11, .54, .1, .5, .751, $\frac{1}{5}$ , $\frac{1}{10}$ ? What pattern can you notice?
Teacher Notes	This can be done as either a whole class activity or half class activity.

•	Have available a range of different size transparent
	containers which can hold water. Fill the containers with a
	range of water levels and place one container between
	groups of four students. Ask students to discuss and agree
	on a numerical value from one to hundred to estimate
	percent 'fullness' of the container. Tell them that they must
	be able to explain and justify their estimate. Repeat the
	activity a number of times using different levels of water
	and giving the students different shape and size containers.
	For the second task have a long and unmarked tape on the
	floor of the classroom to represent the download bar used
	on a cell phone when downloading an app. Use a white
	board marker to mark a point on the tape which indicates
	the level of download reached. Repeat the activity many
	times always ensuring that the mark on the line is above
	10%.
•	Facilitate the students to think of other names they could
	call the percent fullness. Have them notice that we are
	always talking about one whole and part of a whole
	whether we are using fractions, decimals or percentages.
	For example, 50% of the one whole bottle. Encourage
	students to co-ordinate their intuitive understandings of
	percent with strategies for operating on numbers 1-100
	(Strategies such as numerical halving using fingers to
	represent on the container 100, 50, 25, and composition
	100=75+25)
	For the second part facilitate the students to notice that
	numbers are grouped into multiples of powers of tens
	(tens, hundreds, thousands, tenths, hundredths,
	thousandths, and so on).
•	Monitor for students using vocabulary within the language
	of rational numberhalf full, or fifty per cent and that we
	are always talking about out of one hundred.
•	Notice the use of numerical splitting
	used by students to explain and justify the value they have
	put on the fullness of the container.
•	Expect students to represent using a range of different
	representations including justifying using percent,
1	decimals, fractions and pictures of water bottles, chocolate

	<ul> <li>bars, lines. If the students do not use decimals re-represent the measure as equivalent decimals and fractions.</li> <li>Teacher to record in symbols the fractional language students use as they explain. When a half or a quarter or other fractions are used have students re-explain using percent and record as equivalent rational numbers.</li> </ul>
Independent Tasks	<ol> <li>Tan has a 750ml pump bottle. By playtime he has drunk 17% of the bottle. How many ml of water are left in his bottle?</li> <li>Nanaia has a bag of 60 jellybeans that she shares with her friends. She gives 35% to one friend and 25% to another and she keeps the rest? How many of her jellybeans do each of her friends have and how many does she have?</li> <li>Sunny take a 25L container with water on his camping trip. By the end of the first day of camp he has used 25% of the water in the container. How much water has he used (ml/l) how much water is left in the container (ml/l)?</li> <li>Leisi has a bag of 60 jellybeans that she shares with her friends. She gives 15% to one friend and 60% to another and she keeps the rest? How many of her jellybeans do each of her friends have and how many does she have?</li> <li>The cross-country track is 3km long. Jerico runs 65% of the distance before stopping to catch his breath. Max runs <sup>3</sup>/4 of the distance before stopping to catch his breath. How far did each of them run before stopping to catch their breath? Who ran the longest distance before stopping?</li> <li>If you have one glass of water 25% full. How much more water do you need to make it 100% full? What about 10%? 76%? 99%? 10%? 1%? When you add the water to make the glass 100% full have you made more than one whole glass of water? Why or why not?</li> </ol>
Anticipations	

Task 2	You are running on the athletics track and this tape represents the track you run on. If I put the 1-digit card down at the start of it that indicates so far you have run 1 metre and the 2-digit card indicates that you have not reached 2 metres yet. How far have you run exactly? How far have you run now?
	Record using a range of different representations including symbols and be ready to explain and justify how they are equivalent.
Big ideas	A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line. Percent is relative to the size of the whole. A percent is a special type of ratio where a part is compared to a whole and the whole is 100. A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.
Curriculum links	<ul> <li>NA3-1: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.</li> <li>NA3-4: Know how many tenths, tens, hundreds, and thousands are in whole numbers.</li> <li>NA3-5: Know fractions and percentages in everyday use.</li> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals.</li> <li>NA4-5: Know the equivalent decimal and percentage forms for everyday fractions.</li> </ul>
Learning Outcomes: Students will be able to:	<ul> <li>Explain and justify the comparison of a part to the whole.</li> <li>Represent and explain reasoning using corresponding points on a number line.</li> <li>Represent reasoning to explain and justify equivalence using different forms of notation, including symbols and words.</li> </ul>
Mathematical language	Percent, percentage, whole, fraction, fractional number, decimal number, rational number, equal, equivalent
Sharing back/Connect	Select students to share after each estimation who can explain and justify the numerical value that they have agreed on and have represented their estimation in multiple ways. Connect:
	What are the fraction and decimal equivalence for these numbers: 10%, 1%, 5%, 9%, 103%, 901%, 209% If you were showing these to a younger child, what do you think would puzzle them? What would you say in response?

Teacher Notes	• During the launch, revisit equivalent numbers for fractions and decimals.
	<ul> <li>Have available and use the long-unmarked tape on the</li> </ul>
	floor of the classroom to represent a running track used in
	athletics and digit cards to represent whole numbers. Use a
	white board marker to mark a point on the tape which
	indicates where they have reached on the running track
	between numbers. Repeat the activity two times with a
	focus on numbers between numbers. The third time put
	down the 0-digit card at the start and the 1-digit card at the
	end. Mark on the line a place around 5%. Repeat a number
	of times always putting the mark below 10%.
	• Facilitate the students to notice that there are numbers
	between numbers and that includes between 0 and .1.
	Ensure extensive discussion including student explanation
	and justification of why for example 1% is recorded as $\frac{1}{1}$ and 01 and connect students to conscious a constitute
	$\frac{1}{100}$ and .01 and expect students to experience cognitive conflict related to numbers below .1.
	<ul> <li>Expect students to explain and represent using a range of</li> </ul>
	different representations to justify why for example 5% =
	-
	$.05 = \frac{5}{100}$ . These should include water containers,
	chocolate bars and lines but may include a place value
	chart.
	• Notice students who voice cognitive conflict about why
	numbers under 10% are represented as hundredths and
	recorded to three decimal places with a zero to represent
	the tenths.
Independent Tasks	What are their equivalent as percentages, fractions and decimals?
	1. $5\% = =$
	2. $\frac{1}{4} = =$
	31 = =
	4. $\frac{3}{4} = =$
	5. $90\% = =$
	625 = =
	7. Two-thirds = $=$
	8. $\frac{2}{6} = =$
	9. $80\% = =$
	10. Three-tenths = =
Anticipations	
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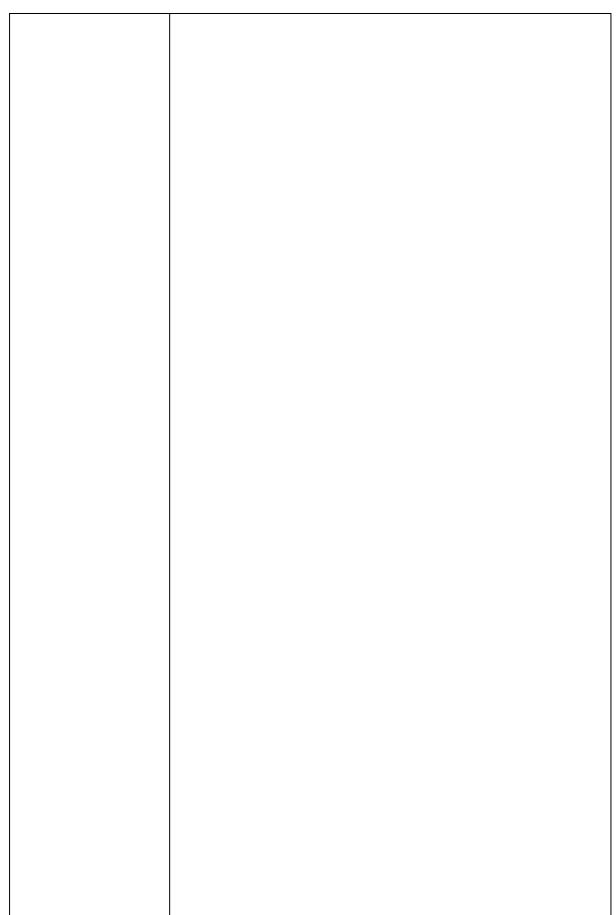
Task 3	Simpson and Mattie were having a jump-off in the sandpit to see where they could jump to if they stood with their toes just before the edge. Josiah measured each jump and he said that Simpson won because although they both jumped 2.38 metres and neither of them reached 2.39 metres Simpson jumped further. Can you record at least 12 different distances for his jump which shows Simpson did jump further. Be ready to explain and justify your answers using number lines, diagrams, drawings, fractions, and decimals.
Big ideas	A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line. A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line. Percent is relative to the size of the whole. A percent is a special type of ratio where a part is compared to a whole and the whole is 100. Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to
	estimate calculations involving fractions and decimals. If two quantities vary proportionally, the quantities are either directly related (as one increases the other increases) or inversely related (as one increases the other decreases).
Curriculum links	<ul> <li>NA3-1: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.</li> <li>NA3-4: Know how many tenths, tens, hundreds, and thousands are in whole numbers.</li> <li>NA3-5: Know fractions and percentages in everyday use.</li> <li>NA3-6: Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality.</li> <li>NA4-2: Understand addition and subtraction of fractions, decimals and integers.</li> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions and decimals.</li> <li>NA4-4: Apply simple linear proportions, including ordering fractions.</li> <li>NA4-5: Know the equivalent decimal and percentage forms for everyday fractions.</li> <li>NA4-6: Know the relative size and place value structure of positive and negative integers and decimals to three places.</li> </ul>
Learning Outcomes:	• Explain and justify the comparison of a part to the whole.
Students will be able to:	• Represent and explain reasoning using corresponding points on a number line.

	• Represent reasoning to explain and justify equivalence using different forms of notation, including symbols and words.		
Mathematical language	Percent, percentage, whole, fraction, fractional number, decimal number, rational number, equal, equivalent		
Sharing back/Connect	Select students to share who have given a range of explanations which cause need for wide student discussion and justification including the use of the place value houses to support reasoning.		
	Connect:		
	What are some numbers between: .5 and .6? 1.124 and 1.125? .0311 and .0412? What do you notice? Can you make a conjecture about numbers between numbers?		
Teacher Notes	<ul> <li>During the launch revisit equivalences between fractions and decimals to two decimal places</li> <li>Have place value houses (which include decimal places) displayed on the wall but do not direct students' attention to it until the sharing back. See place value house with the overlap of decimals with the ones place holder</li> <li>Th H T O Tth Hth Then TThe HThen</li> <li>Explore the notion that places to the left of the decimal point are powers of ten. 10 = 10<sup>1</sup> 100 = 10<sup>2</sup></li> <li>The place values to the right of the decimal place are also powers of ten: .1 = 10<sup>-1</sup> .01 = 10<sup>-2</sup> .001 = 10<sup>-2</sup></li> <li>When numbers are written with decimal notation, the relationship between the places to the right of the decimal point is the same as the relationship between the left of the decimal point is the students to notice that you are always talking about a fraction or decimal of one whole and therefore in the place value houses the decimal dot does not separate</li> </ul>		

	the ones house from the tenths but rather overlaps across
	it. • Notice students who recognize that there are infinite
	• Notice students who recognise that there are infinite numbers between numbers.
	Expect students to represent using symbols and in the end
	to the place in the place value houses.
Independent Tasks	What are the fraction, percentage, or decimal equivalences for the
	following?
	1. 30%
	2. 12%
	3. $\frac{1}{2}$
	4. 33%
	590 618
	767
	81
	93
Anticipations	1075
miciputions	

Task 4	There are three finalists in a gymnastics competition. Here are			on. Here are		
	their scores	T				
	Contestant	Floor	Bar	Vault	Beam	
	1	8.903	7.96	8.897	9.03	
	2	9.1	7.991	7.98	9.004	
	3	7.567	7.99	8.0	9.039	
	Who came fi	rst? Second	l? Third?			
	How many p	oints would	d the second f	inalist have n	eeded to come	
	first?					
	How many p	oints would	d the third fin	alist have nee	ded to come	
	first?					
Big ideas	A decimal is another name for a fraction and thus can be				can be	
	associated w	ith the corre	esponding po	int on the num	nber line.	
	A percent is	another way	y to write a d	ecimal that co	ompares part to	
	a whole when	re the whole	e is 100 and t	hus can be as	sociated with	
	the correspon	nding point	on the number	er line.		
	Percent is rel	ative to the	size of the w	hole. A perce	ent is a special	
	type of ratio 100.	where a pai	rt is compared	d to a whole a	nd the whole is	
	Benchmark f	Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to				
	estimate calculations involving fractions and decimals. If two quantities vary proportionally, the quantities are either directly related (as one increases the other increases) or inversely related (as one increases the other decreases). The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers.					
Curriculum links	<b>NA3-1:</b> Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and					
	percentages.					
	<ul> <li>NA3-4: Know how many tenths, tens, hundreds, and thousands are in whole numbers.</li> <li>NA3-5: Know fractions and percentages in everyday use.</li> <li>NA3-6: Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality.</li> </ul>					
	<ul> <li>NA4-2: Understand addition and subtraction of fractions, decimals and integers.</li> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions and decimals.</li> </ul>					
					,	
					of amounts	
	<b>NA4-4:</b> Apply simple linear proportions, including ordering					
	fractions.			_		
		-	alent decima	l and percenta	age forms for	
	everyday fra				_	
			-	lace value stru		
<b>.</b>				cimals to thre		
Learning Outcomes:		-		lving numbers	-	
Students will be able	decin	nal places a	nd explain an	d justify the s	solutions.	
to:						

	-	ning to explain and jus ers up to three decimal	
	)	-	-
Mathematical	Percent, percentage, whole, fraction, fractional number, decimal		
language	number, rational number, equal, equivalent		
Sharing	Select students to share		-
back/Connect	their reasoning and con		-
	explain and justify why	they used numbers in	groupings of ten.
	Connect:		
	Represent these on a nu	umberline	
	.9 + .1 = 1		
	.09 + .01 = .1		
	.009 + .001 = .01		
	What patterns do you n	otice? Be prepared to e	xplain and justify
	the pattern you notice u		
Teacher Notes	1	ch, revisit the use of a 1	
	•	ole numbers. Use num	
		t 10, 100, and 1000. Ex	
		ey can relate these to p	-
	numbers.	- J	
		lue house for whole an	d decimal numbers
	on the wall.		
		idents to notice that wh	en recombining
		the next hundredth, t	-
		e value chart is from rig	
		whole numbers with a	
	for subtraction.	whole humbers with a	indicining process
		who use recombining t	he decimal numbers
	• Notice students who use recombining the decimal numbers in groupings of ten.		
	0 1 0		ning using informal
	• Expect students to represent their reasoning using informal notation across the page and on a numberline and <b>not</b>		
		algorithm where the	
		the decimal dot.	up •== •
Independent Tasks	Gina had to do some ho		it some decimal
T. T	numbers in order from	-	
	.90146 .901	-	.70000
	.4405 .321		.5
	.450000 .451		.52
	0.901 0.4		.520
	You need to put them in		her out and then
	write her an explanation	• •	
	she had them in. Explai	• •	-
	row.	2	C
	10111		
Anticipations			



Task 5	Solve these equations using two different ways
	9.705 + 7.99
	8.095 + 9.91
	5.5 + 6.5123
	7.3 - 0.27
	8.109 - 1.09
	1.45 - 0.55
Big ideas	A decimal is another name for a fraction and thus can be
Dig lucus	associated with the corresponding point on the number line.
	A percent is another way to write a decimal that compares part to
	a whole where the whole is 100 and thus can be associated with
	the corresponding point on the number line. Percent is relative to the size of the whole.
	A percent is a special type of ratio where a part is compared to a
	whole and the whole is 100.
	Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to
	estimate calculations involving fractions and decimals.
	If two quantities vary proportionally, the quantities are either
	directly related (as one increases the other increases) or
	inversely related (as one increases the other decreases).
	The effects of operations for addition and subtraction with
	fractions and decimals are the same as those with whole numbers.
Curriculum links	<b>NA3-1:</b> Use a range of additive and simple multiplicative
	strategies with whole numbers, fractions, decimals, and
	percentages.
	<b>NA3-4:</b> Know how many tenths, tens, hundreds, and thousands
	are in whole numbers.
	NA3-5: Know fractions and percentages in everyday use.
	<b>NA3-6:</b> Record and interpret additive and simple multiplicative
	strategies, using words, diagrams, and symbols, with an
	understanding of equality.
	NA4-2: Understand addition and subtraction of fractions,
	decimals and integers.
	NA4-3: Find fractions, decimals, and percentages of amounts
	expressed as whole numbers, simple fractions and decimals.
	<b>NA4-4:</b> Apply simple linear proportions, including ordering
	fractions.
	<b>NA4-5:</b> Know the equivalent decimal and percentage forms for
	everyday fractions.
	<b>NA4-6:</b> Know the relative size and place value structure of
Learning Outcomes:	positive and negative integers and decimals to three places.
Students will be able	• Solve problems involving decimal by adding or
to:	subtracting and explain and justify the solution.
ເບ.	• Represent reasoning to explain and justify place value
	involving decimal numbers.
	• Explain and justify reasoning using notation, symbols, and
	words.

Mathematical language	Percent, percentage, whole, fraction, fractional number, decimal number, rational number, equal, equivalent		
Sharing back/Connect	Select students to share who can explain and justify their solution drawing on a range of representations and using place value to justify what they did.		
	Connect:		
	What pattern can you see? .101 .1001 .10001 Be ready to explain and justify the pattern you see using place value.		
Teacher Notes	<ul> <li>During the launch, continue to explore the face, place, and total value of whole and decimal numbers.</li> <li>Facilitate the students to notice that when subtracting using decimals they need to be recombined to make the next hundredth, tenth or one and that the shift in the place value chart is from right to left the same as when adding whole numbers.</li> <li>Notice students who use recombining the decimal numbers in groupings of ten.</li> <li>Expect students to represent their reasoning using informal notation across the page and on a numberline and not using a formal algorithm where they line up the numbers after the decimal dot.</li> </ul>		
Independent Tasks	True or false? 3.15 + 3.15 = 3.3 1.9 + 1.9 = 1.18 1.09 + 1.009 = 2.099 0.25 + 1.85 = 1.10 2.09 + 1.11 = 3.110		
Anticipations	Use place value to explain and justify your reasoning.		

Task 6	<ul> <li>Sima is going on a holiday to Samoa. Different family members give him money to spend, and he has some money saved up. The exchange rate is \$1 New Zealand for 1.6557 Samoan tala.</li> <li>His uncle gives him NZ\$10. His aunty gives him NZ\$100 and his father gives him NZ\$185. How much Samoan tala will he get in exchange?</li> <li>If he was going to the U.S., the exchange rate for \$1 New Zealand is USD\$.63. How much would he get with the same amount of money in U.S. currency?</li> </ul>
Big ideas	A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line. A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line. Percent is relative to the size of the whole. A percent is a special type of ratio where a part is compared to a whole and the whole is 100. Different real-world interpretations can be associated with the product of a whole number and fraction (decimal), a fraction (decimal) and whole number, and a fraction and fraction (decimal and decimal). Different real-world interpretations can be associated with division calculations involving fractions (decimals). The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers. Division with a decimal divisor is changed to an equivalent calculation with a whole number divisor by multiplying the divisor and dividend by an appropriate power of ten. Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to estimate calculations involving fractions and decimals.
Curriculum links	<ul> <li>NA3-1: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.</li> <li>NA3-4: Know how many tenths, tens, hundreds, and thousands are in whole numbers.</li> <li>NA3-5: Know fractions and percentages in everyday use.</li> <li>NA3-6: Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality.</li> <li>NA4-2: Understand addition and subtraction of fractions, decimals and integers.</li> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions and decimals.</li> </ul>

	<b>NA4-4:</b> Apply simple linear proportions, including ordering fractions.
	<b>NA4-5:</b> Know the equivalent decimal and percentage forms for
	everyday fractions.
	<b>NA4-6:</b> Know the relative size and place value structure of
	positive and negative integers and decimals to three places.
Learning Outcomes:	• Explain and justify the comparison of a part to the whole.
Students will be able	• Represent reasoning using different forms of notation,
to:	including symbols and words.
	• Solve problems involving decimal numbers by multiplying
	and explain and justify the solution.
	• Represent reasoning to explain and justify place value
	involving decimal numbers.
Mathematical	Percent, percentage, whole, fraction, fractional number, decimal
language	number, rational number, equal, equivalent
Sharing	Select students to share who are able to explain and justify their
back/Connect	reasoning using groupings (powers) of tens and hundreds.
	Connect:
	As you solve these, think about their place on the place value
	chart.
	$6 \ge 1 = 6 \ge 10 = 10$
	$6 \times 10^{-1}$ 6 x 100 =
	$6 \times 100^{-1}$
	$.6 \times 10 =$
	$.6 \ge 100 =$
	Can you identify and explain the pattern you notice in the shifts in
	place value?
Teacher Notes	• Facilitate the students to notice that when multiplying
	decimals, you are using powers of ten.
	• Expect students to represent using place value and
	symbols.
Independent Tasks	Solve the following:
	.5 + .05 =
	.5 + .505 =
	.3 + .03 =
	.3 + .0303 =
	.7 + .07 =
	.7 + 0.707 =
	Write a statement to explain your solution to a younger child
	using place value.

Anticipations	

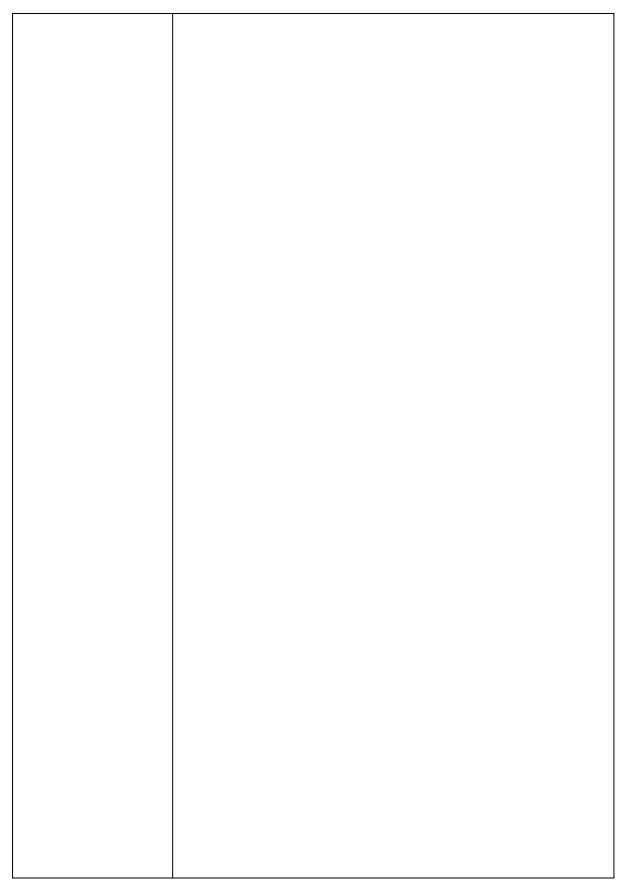
Task 7	Where does the decimal go?
	24 x 63
	0.24 x 6.3
	24 x 0.63
	2.4 x 63
	0.24 x 0.63
	0.24 X 0.05
	In your group decide where you will put the decimal in each
	equation and be ready to explain and justify why you put it where
	you did.
	you ulu.
	When you have put the decimal in each solution complete the
	equation to check whether your reasoning was correct. Be ready
	to explain why you needed to change your solution.
<b>Big ideas</b>	A decimal is another name for a fraction and thus can be
Dig lucas	associated with the corresponding point on the number line.
	A percent is another way to write a decimal that compares part to
	a whole where the whole is 100 and thus can be associated with
	the corresponding point on the number line.
	Percent is relative to the size of the whole.
	A percent is a special type of ratio where a part is compared to a
	whole and the whole is 100.
	Different real-world interpretations can be associated with the
	product of a whole number and fraction (decimal), a fraction
	(decimal) and whole number, and a fraction and fraction (decimal
	and decimal).
	Different real-world interpretations can be associated with
	division calculations involving fractions (decimals).
	The effects of operations for addition and subtraction with
	fractions and decimals are the same as those with whole numbers.
	Division with a decimal divisor is changed to an equivalent
	calculation with a whole number divisor by multiplying the
	divisor and dividend by an appropriate power of ten.
	Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to
	estimate calculations involving fractions and decimals.
Curriculum links	<b>NA3-1:</b> Use a range of additive and simple multiplicative
	strategies with whole numbers, fractions, decimals, and
	percentages.
	NA3-4: Know how many tenths, tens, hundreds, and thousands
	are in whole numbers.
	NA3-5: Know fractions and percentages in everyday use.
	<b>NA3-6:</b> Record and interpret additive and simple multiplicative
	strategies, using words, diagrams, and symbols, with an
	understanding of equality.
	NA4-2: Understand addition and subtraction of fractions,
	decimals and integers.

	<ul> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions and decimals.</li> <li>NA4-4: Apply simple linear proportions, including ordering fractions.</li> <li>NA4-5: Know the equivalent decimal and percentage forms for</li> </ul>
	everyday fractions.
	<b>NA4-6:</b> Know the relative size and place value structure of
	positive and negative integers and decimals to three places.
Learning Outcomes:	Represent reasoning using different forms of notation,
Students will be able	including symbols and words.
to:	• Solve problems involving decimal numbers by multiplying
	and explain and justify the solution.
	• Represent reasoning to explain and justify place value
	involving decimal numbers.
Mathematical	Percent, percentage, whole, fraction, fractional number, decimal
language	number, rational number, equal, equivalent
Sharing	Select students to share who are able to explain and justify their
back/Connect	reasoning using a range of representations including place value
	and notation
	Connect:
	Solve these using their fraction equivalents and then rename them
	as decimals and show the result in a place value chart: 3.2 x 1.1
	1.3 x 2.5
	1.2 x .1.2
	What pattern can you identify? Can you explain why?
Teacher Notes	• During the launch, explore the place value related to
	multiplication and division of whole numbers, and
	fractional numbers
	• Facilitate the students to notice that when multiplying a
	rational number by a rational number that they need to
	convert the decimals to their fraction equivalents. In the
	connect ensure that discussion relates to the way in which
	changing the decimals to fractions and then back to
	decimals corresponds to moving the decimal two places to
Indonandant Tasks	the left.
Independent Tasks	1. Consider these two calculations $2^{1} \times 2^{1}$ and $2.276 \times 2.18$ With out doing the colorlations
	$3\frac{1}{2} \times 2\frac{1}{4}$ and 2.276 x 3.18 Without doing the calculations
	which product do you think would be the larger? First
	provide a reason for your answer and then check your
	reasoning by doing the calculations.
	2. Without doing the calculations record how much larger is 0.76 x 5 than 0.75 x 5? First provide a reason for your
	0.70 x 5 than 0.75 x 5; Thist provide a reason for your

	<ul> <li>answer and then check your reasoning by doing the calculations.</li> <li>3. Write these decimals as fractions <ul> <li>.6 3.11 0.234 0.04 7.39</li> </ul> </li> <li>4. Write these fractions as decimals <ul> <li>1/10 15/10 5/1000 264/1000 8/100</li> </ul> </li> <li>5. Illustrate these decimals on a number line <ul> <li>0.2 0.6 1.7 2.5 7.6 5 3.1</li> </ul> </li> </ul>
Anticipations	

Task 8	The bus trip to Whangarei was 282km. It took exactly 4.5 hours to
	travel. What was the average rate in kilometres per hour?
	Use the same strategy you used to solve this equation. Estimate
	first and then do the calculation.
	45.7 ÷ 1.83
	Be ready to explain and justify your reasoning.
Big ideas	A decimal is another name for a fraction and thus can be
	associated with the corresponding point on the number line.
	A percent is another way to write a decimal that compares part to
	a whole where the whole is 100 and thus can be associated with
	the corresponding point on the number line.
	Percent is relative to the size of the whole.
	A percent is a special type of ratio where a part is compared to a whole and the whole is 100.
	Different real-world interpretations can be associated with the
	product of a whole number and fraction (decimal), a fraction
	(decimal) and whole number, and a fraction and fraction (decimal)
	and decimal).
	Different real-world interpretations can be associated with
	division calculations involving fractions (decimals).
	The effects of operations for addition and subtraction with
	fractions and decimals are the same as those with whole numbers.
	Division with a decimal divisor is changed to an equivalent
	calculation with a whole number divisor by multiplying the
	divisor and dividend by an appropriate power of ten.
	Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to
	estimate calculations involving fractions and decimals.
	A ratio is a multiplicative comparison of quantities; there are
	different types of comparisons that can be represented as ratios.
	Ratios give the relative sizes of the quantities being compared, not
	necessarily the actual sizes.
	Ratios can be expressed as units by finding an equivalent ratio
	where the second term is one.
Curriculum links	<b>NA3-1:</b> Use a range of additive and simple multiplicative
	strategies with whole numbers, fractions, decimals, and
	percentages. <b>NA3-4:</b> Know how many tenths, tens, hundreds, and thousands
	are in whole numbers.
	NA3-5: Know fractions and percentages in everyday use.
	NA3-6: Record and interpret additive and simple multiplicative
	strategies, using words, diagrams, and symbols, with an
	understanding of equality.
	NA4-2: Understand addition and subtraction of fractions,
	decimals and integers.
	<b>NA4-3:</b> Find fractions, decimals, and percentages of amounts
	expressed as whole numbers, simple fractions and decimals.

Learning Outcomes: Students will be able to:	<ul> <li>NA4-4: Apply simple linear proportions, including ordering fractions.</li> <li>NA4-5: Know the equivalent decimal and percentage forms for everyday fractions.</li> <li>NA4-6: Know the relative size and place value structure of positive and negative integers and decimals to three places.</li> <li>Find equivalent rates.</li> <li>Represent reasoning using different forms of notation, including symbols and words.</li> <li>Use multiplicative understanding of place value to solve multiplication and division problems with decimal numbers.</li> <li>Represent reasoning to explain and justify place value involving decimal numbers.</li> </ul>
	myorying deemai numbers.
Mathematical	Percent, percentage, whole, fraction, fractional number, decimal
language	number, rational number, equal, equivalent
Sharing back/Connect	Select students to share who are able to explain and justify their reasoning using a range of ways to represent their reasoning.
	<b>Connect:</b> 146 $\div$ 7 = 20857 correct to five digits but without the decimal point. Can you use only this information to solve the following? 146 $\div$ 0.7, 1.46 $\div$ 7, 14.6 $\div$ 0.7, 1460 $\div$ 70 Can you explain the patterns you notice using place value?
Teacher Notes	<ul> <li>Facilitate the students to notice the need to consider use of multiplication and rounding to 4 or 5 and 280.</li> <li>Notice students in the second problem who round to 46 and 2 before solving.</li> </ul>
Independent Tasks	Solve these using their fraction equivalents and then rename them as decimals and show the result in a place value chart: 5.3 x 1.1 1.4 x 3.5 1.2 x .1.2 1.7 x 1.3 2.5 x 2.5 What pattern can you identify? Can you explain why?
Anticipations	



Task 9	Our local pizza restaurant makes a large quantity of dough which they cool store ready for use, using 36 cups of flour. The ratio of cups of flour to cups of water they use is 9:4. How much water should they use? What if they use 50 cups of flour? The ratio of cups of flour to cups of water they use is 10:5. How much water should they use?
	What if they use 66 cups of flour? The ratio of cups of flour to cups of water they use is 12:7. How much water should they use?
Big ideas	A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line. A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line. Percent is relative to the size of the whole. A percent is a special type of ratio where a part is compared to a whole and the whole is 100. Different real-world interpretations can be associated with the product of a whole number, and a fraction (decimal), a fraction (decimal) and whole number, and a fraction and fraction (decimal and decimal). Different real-world interpretations can be associated with division calculations involving fractions (decimals). The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers. Division with a decimal divisor is changed to an equivalent calculation with a whole number divisor by multiplying the divisor and dividend by an appropriate power of ten. Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to estimate calculations involving fractions and decimals. A ratio is a multiplicative comparison of quantities; there are different types of comparisons that can be represented as ratios. Ratios give the relative sizes of the quantities being compared, not necessarily the actual sizes. Ratios can be expressed as units by finding an equivalent ratio
Curriculum links	where the second term is one.NA3-1: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and
	<ul> <li>percentages.</li> <li>NA3-4: Know how many tenths, tens, hundreds, and thousands are in whole numbers.</li> <li>NA3-5: Know fractions and percentages in everyday use.</li> <li>NA3-6: Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality.</li> </ul>

Learning Outcomes: Students will be able to:	<ul> <li>NA4-2: Understand addition and subtraction of fractions, decimals and integers.</li> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions and decimals.</li> <li>NA4-4: Apply simple linear proportions, including ordering fractions.</li> <li>NA4-5: Know the equivalent decimal and percentage forms for everyday fractions.</li> <li>NA4-6: Know the relative size and place value structure of positive and negative integers and decimals to three places.</li> <li>Find equivalent ratios.</li> <li>Represent reasoning using different forms of notation, including symbols and words.</li> <li>Use multiplicative understanding of place value to solve multiplication and division problems with decimal numbers.</li> <li>Represent reasoning to explain and justify place value involving decimal numbers.</li> </ul>
Mathematical	Percent, percentage, whole, fraction, fractional number, decimal
language	number, rational number, equal, equivalent, ratio
Sharing back/Connect	Select students to share who are able to explain and justify their
Dack/Connect	reasoning using a range of ways to represent their reasoning.
	Connect:
	The ratio of girls to boys is 5:6. What is the fraction of girls to boys?
	The ratio of girls to boys is 10:20. What is the fraction of girls to boys?
	The ratio of girls to boys is 1:2. What is the fraction of girls to boys?
	What pattern can you see? Can you make a conjecture about what you discovered?
Teacher Notes	<ul> <li>Facilitate the students to notice the need to keep a balance of both sides in a clear representation</li> <li>Facilitate students to understand that when considering ratios, the relationship between two quantities is invariant across these situations.</li> <li>Expect students to represent using a systematic way of recording their reasoning as ratios</li> </ul>
Independent Tasks	
	$146 \div 7 = 20857$ is correct to five digits but without the decimal point. Can you use only this information to solve the following? $146 \div 0.7, 1.46 \div 7, 14.6 \div 0.7, 1460 \div 70$ Can you explain the patterns you notice using place value?

Anticipations	

Task 10	
	Raj's older brother is saving for a new bike which will cost \$480. He earns \$1500 per month. He spends his money on bills, food
	and extras in the ratio of 8:3:4. Of the money he spends on extras,
	he spends 80% and puts 20% into his savings account. How long
D· · · I	will it take his brother to save for his new bike?
Big ideas	A decimal is another name for a fraction and thus can be
	associated with the corresponding point on the number line.
	A percent is another way to write a decimal that compares part to
	a whole where the whole is 100 and thus can be associated with
	the corresponding point on the number line.
	Percent is relative to the size of the whole.
	A percent is a special type of ratio where a part is compared to a
	whole and the whole is 100.
	Different real-world interpretations can be associated with the
	product of a whole number and fraction (decimal), a fraction
	(decimal) and whole number, and a fraction and fraction (decimal
	and decimal).
	Different real-world interpretations can be associated with
	division calculations involving fractions (decimals).
	The effects of operations for addition and subtraction with
	fractions and decimals are the same as those with whole numbers.
	Division with a decimal divisor is changed to an equivalent
	calculation with a whole number divisor by multiplying the
	divisor and dividend by an appropriate power of ten.
	Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to
	estimate calculations involving fractions and decimals.
	A ratio is a multiplicative comparison of quantities; there are
	different types of comparisons that can be represented as ratios.
	Ratios give the relative sizes of the quantities being compared, not
Curriculum links	
	are in whole numbers.
	<b>NA3-5:</b> Know fractions and percentages in everyday use.
	understanding of equality.
	NA4-2: Understand addition and subtraction of fractions,
	decimals and integers.
	e
	expressed as whole numbers, simple fractions and decimals.
	fractions.
Curriculum links	<ul> <li>estimate calculations involving fractions and decimals.</li> <li>A ratio is a multiplicative comparison of quantities; there are different types of comparisons that can be represented as ratios. Ratios give the relative sizes of the quantities being compared, r necessarily the actual sizes.</li> <li>Ratios can be expressed as units by finding an equivalent ratio where the second term is one.</li> <li>NA3-1: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.</li> <li>NA3-4: Know how many tenths, tens, hundreds, and thousands are in whole numbers.</li> <li>NA3-5: Know fractions and percentages in everyday use.</li> <li>NA3-6: Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality.</li> <li>NA4-2: Understand addition and subtraction of fractions, decimals and integers.</li> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions and decimals.</li> <li>NA4-4: Apply simple linear proportions, including ordering</li> </ul>

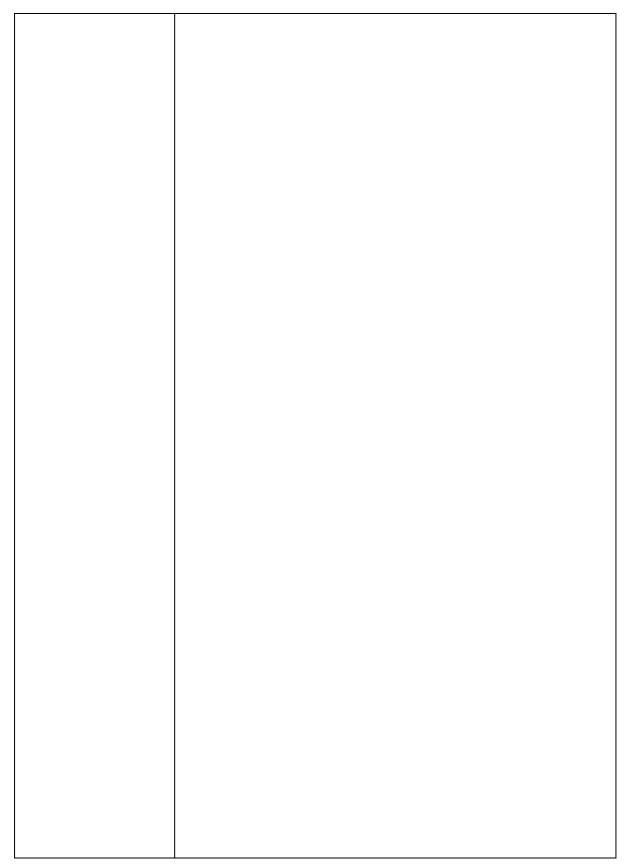
Learning Outcomes: Students will be able to:	<ul> <li>NA4-5: Know the equivalent decimal and percentage forms for everyday fractions.</li> <li>NA4-6: Know the relative size and place value structure of positive and negative integers and decimals to three places.</li> <li>Find equivalent ratios.</li> <li>Represent reasoning using different forms of notation, including symbols and words.</li> <li>Use multiplicative understanding of place value to solve multiplication and division problems with decimal numbers.</li> <li>Represent reasoning to explain and justify place value involving decimal numbers.</li> </ul>
Mathematical language	Percent, percentage, whole, fraction, fractional number, decimal number, rational number, equal, equivalent
Sharing back/Connect	Select students to share who are able to explain and justify their reasoning using a range of ways to represent their reasoning.
	<b>Connect:</b> On a farm the ratio of pigs to goats is 4:1. The ratio of pigs to piglets is 1:6 and the ratio of goats to kids is 1:2. What fraction of the animals on the farm are babies?
Teacher Notes	<ul> <li>Facilitate the students to notice the need to keep a balance of both sides in a clear representation</li> <li>Notice students who use informal reasoning before formal reasoning</li> <li>Expect students to represent using a systematic way of recording their reasoning as ratios</li> </ul>
Independent Tasks	The ratio of girls to boys is 6:8. What is the fraction of girls to boys? The ratio of girls to boys is 10:30. What is the fraction of girls to boys? The ratio of girls to boys is 2:3. What is the fraction of girls to boys? What pattern can you see? Can you make a conjecture about what you discovered? Record your reasoning.
Anticipations	

Task 11 (optional)	Replace the letter with a number
	$m \ge 10 = 35$
	$j \ge 1.46$
	$346 = p \times 10 + y \times 100$
	$3.4 = 34 \div a$
	$b = .77 \div 100$
	Write five more to share with the group.
Big ideas	A decimal is another name for a fraction and thus can be
	associated with the corresponding point on the number line.
	A percent is another way to write a decimal that compares part to
	a whole where the whole is 100 and thus can be associated with
	the corresponding point on the number line.
	Percent is relative to the size of the whole.
	A percent is a special type of ratio where a part is compared to a
	whole and the whole is 100.
	Different real-world interpretations can be associated with the
	product of a whole number and fraction (decimal), a fraction
	(decimal) and whole number, and a fraction and fraction (decimal
	and decimal).
	Different real-world interpretations can be associated with
	division calculations involving fractions (decimals).
	The effects of operations for addition and subtraction with
	fractions and decimals are the same as those with whole numbers.
	Division with a decimal divisor is changed to an equivalent
	calculation with a whole number divisor by multiplying the
	divisor and dividend by an appropriate power of ten.
	Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to
	estimate calculations involving fractions and decimals.
	A ratio is a multiplicative comparison of quantities; there are
	different types of comparisons that can be represented as ratios.
	Ratios give the relative sizes of the quantities being compared, not
	necessarily the actual sizes.
	Ratios can be expressed as units by finding an equivalent ratio
	where the second term is one.
Curriculum links	NA3-1: Use a range of additive and simple multiplicative
	strategies with whole numbers, fractions, decimals, and
	percentages.
	NA3-4: Know how many tenths, tens, hundreds, and thousands
	are in whole numbers.
	NA3-5: Know fractions and percentages in everyday use.
	<b>NA3-6:</b> Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an
	understanding of equality.
	<b>NA4-2:</b> Understand addition and subtraction of fractions,
	decimals and integers.
	<b>NA4-3:</b> Find fractions, decimals, and percentages of amounts
	expressed as whole numbers, simple fractions and decimals.

	<b>NA4-4:</b> Apply simple linear proportions, including ordering
	fractions.
	<b>NA4-5:</b> Know the equivalent decimal and percentage forms for
	everyday fractions.
	<b>NA4-6:</b> Know the relative size and place value structure of
	positive and negative integers and decimals to three places.
Learning Outcomes:	• Represent reasoning using different forms of notation,
Students will be able	including symbols and words.
to:	• Use multiplicative understanding of place value to solve
	multiplication and division problems with decimal
	numbers.
	<ul> <li>Represent reasoning to explain and justify place value</li> </ul>
	involving decimal numbers.
Mathematical	Percent, percentage, whole, fraction, fractional number, decimal
language	number, rational number, equal, equivalent
Sharing	Select students to share who are able to explain and justify their
back/Connect	reasoning using a range of ways to represent their reasoning.
	Connect:
	Share those constructed by the students testing out why they work
	as they do. Focus the student reasoning on using multiplicative
	understanding of place value
<b>Teacher Notes</b>	• Notice students who use relationships in their solutions to
	explain and justify
	• Expect students to represent using a systematic way of
	recording their reasoning as ratios
Independent Tasks	Where does the decimal go? Before you compute an answer put in
	the decimal and write an explanation of why you put it where it is.
	0.24 x 6.3
	24 x 0.63
	2.4 x 63
	0.24 x 0.63
	Now check your answers with computation. If there are
	differences record your reasoning.
Anticipations	

Task 12 (antional)	Replace the letter with a number
Task 12 (optional)	67.718 + w = 91.381
	3.0009 + 0.111 = x
	8.9106 = 10 - y
	34  x r = 68000
	.02 = s x .2
	.32 = t x .2
	2.4  x u = 48
	Can you write some using division to share with the group?
Big ideas	A decimal is another name for a fraction and thus can be
	associated with the corresponding point on the number line.
	A percent is another way to write a decimal that compares part to
	a whole where the whole is 100 and thus can be associated with
	the corresponding point on the number line.
	Percent is relative to the size of the whole.
	A percent is a special type of ratio where a part is compared to a
	whole and the whole is 100.
	Different real-world interpretations can be associated with the
	product of a whole number and fraction (decimal), a fraction
	(decimal) and whole number, and a fraction and fraction (decimal
	and decimal).
	Different real-world interpretations can be associated with
	division calculations involving fractions (decimals).
	The effects of operations for addition and subtraction with
	fractions and decimals are the same as those with whole numbers.
	Division with a decimal divisor is changed to an equivalent
	<b>-</b>
	calculation with a whole number divisor by multiplying the
	divisor and dividend by an appropriate power of ten.
	Benchmark fractions like $\frac{1}{2}(0.5)$ and $\frac{1}{4}(0.25)$ can be used to
	estimate calculations involving fractions and decimals.
	A ratio is a multiplicative comparison of quantities; there are
	different types of comparisons that can be represented as ratios.
	Ratios give the relative sizes of the quantities being compared, not
	necessarily the actual sizes.
	Ratios can be expressed as units by finding an equivalent ratio
	where the second term is one.
Curriculum links	<b>NA3-1:</b> Use a range of additive and simple multiplicative
	strategies with whole numbers, fractions, decimals, and
	percentages.
	<b>NA3-4:</b> Know how many tenths, tens, hundreds, and thousands
	are in whole numbers.
	NA3-5: Know fractions and percentages in everyday use.
	<b>NA3-6:</b> Record and interpret additive and simple multiplicative
	strategies, using words, diagrams, and symbols, with an
	understanding of equality.
	NA4-2: Understand addition and subtraction of fractions,
	decimals and integers.

Learning Outcomes: Students will be able to:	<ul> <li>NA4-3: Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions and decimals.</li> <li>NA4-4: Apply simple linear proportions, including ordering fractions.</li> <li>NA4-5: Know the equivalent decimal and percentage forms for everyday fractions.</li> <li>NA4-6: Know the relative size and place value structure of positive and negative integers and decimals to three places.</li> <li>Represent reasoning using different forms of notation, including symbols and words.</li> <li>Use multiplicative understanding of place value to solve</li> </ul>
	<ul> <li>Ose multiplicative understanding of place value to solve multiplication and division problems with decimal numbers.</li> <li>Represent reasoning to explain and justify place value involving decimal numbers.</li> </ul>
Mathematical	Percent, percentage, whole, fraction, fractional number, decimal
language	number, rational number, equal, equivalent
Sharing	Select students to share who are able to explain and justify their
back/Connect	reasoning using a range of ways to represent their reasoning.
	Connect:
	Share those constructed by the students testing out why they work as they do
<b>Teacher Notes</b>	• Notice students who use relationships in their solutions to
	explain and justify
	• Expect students to represent using a systematic way of
L. J J 4 Tl	recording their reasoning as ratios
Independent Tasks	Where does the decimal go? Before you compute an answer put in the decimal and write an explanation of why you put it where it is.
	0.24 x 6.3
	24 x 0.63
	2.4 x 63
	0.24 x 0.63
	Now check your answers with computation. If there are differences record your reasoning. <u>OR</u>
	Select one or more of the following assessment tasks (attached at the end of the document) as the independent activity: NR7: Decimals
	NR9: Fractions, Decimals, Percentages
Anticipations	





NUMBER – FRACTIONS/PROPORTION, RATIOS & DECIMALS: LEVEL 3 - 4 Task NR7

Give 12 examples of different sized decimal numbers. Put them in order from smallest to largest. Prove that they are in the correct order by using three different representations.



NUMBER – FRACTIONS/PROPORTION, RATIOS & DECIMALS: LEVEL 3 - 5 TASK NR9

Write some word problems for a friend involving any of the operations (addition, subtraction, multiplication, division) using fractions, decimals, or percentages. Show how you would solve the problems.